Code: 13A04304

R13

B.Tech II Year I Semester (R13) Regular & Supplementary Examinations December 2015

PROBABILITY THEORY & STOCHASTIC PROCESSES

(Electronics and Communication Engineering)

Time: 3 hours | Max. Marks: 70

PART – A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) What are the conditions to be satisfied for the statistical independence of three events A, B and C?

(b) Show that $P(x_1 < X \leq x_2) = F_X(x_2) - F_X(x_1)$.

(c) Two random variables X and Y have the following values:

$E[X] = E[Y] = \frac{7}{12}, E[XY] = \frac{1}{3}$ and $\sigma_X = \sigma_Y = \sqrt{\frac{11}{144}}$. Find the correlation coefficient.

(d) Define the joint moments about the origin.

(e) Define WSS random process.

(f) Determine the mean-square value of a random process with autocorrelation function: $R_{XX}(\tau) = e^{-|\tau|}$.

(g) A random process has the power density spectrum $S_{XX}(\omega) = \frac{6\omega^2}{1+\omega^4}$. Find the average power in the process.

(h) Define rms bandwidth of the power spectrum.

(i) Impulse response of a linear system is $h(t) = [1-t][u(t) - u(t-1)]$. The input to this system is a sample function from a random process having an autocorrelation function of $R_{XX}(\tau) = \delta(\tau)$. Find the autocorrelation of the output.

(j) A stationary random process with a mean of 2 is passed through an LTI system with $h(t) = 2e^{-2t}\,u(t)$. Determine the mean of the output process.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

2 (a) Define conditional distribution and density functions and list their properties.

(b) A continuous random variable X has a PDF $f_X(x) = 3x^2,\ 0 \leq x \leq 1$. Find ‘a’ and ‘b’ such that:
(i) $P\{X \leq a\} = P\{X > a\}$. (ii) $P\{X > b\} = 0.05$.

OR

3 (a) Define random variable and give the concept of random variable with an example.

(b) The probability density function of a random variable has the form $f_X(x) = 5e^{-kx}u(x)$, where $u(x)$ is the unit step function. Find the probability that $X > 1$.

UNIT – II

4 (a) Define marginal density and distribution functions.

(b) Let X and Y be jointly continuous random variables with joint probability density function:

$$f_{XY}(x,y) = x^2 + \frac{xy}{3}; \quad 0 \leq x \leq 1, 0 \leq y \leq 2$$
$$= 0 \quad ; \quad elsewhere$$

Find: (i) $f_X(x)$. (ii) $f_Y(y)$. (iii) Are X and Y independent?

OR

5 (a) State central limit theorem for the following two cases:
(i) Equal distributions. (ii) Unequal distributions.

(b) Let $f_{XY}(x,y) = 4x + 2y,\ 0 \leq x \leq 1/2,\ 0 \leq y \leq 1$ and zero elsewhere. Find $P\{X \leq 1/4\}$.

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UNIT – III

6 (a) A random process has sample functions of the form: $X(t) = \begin{cases} A; & 0 \le t \le 1 \\ 0; & else\ where \end{cases}$

Where A is a random variable uniformly distributed from 0 to 10. Find the autocorrelation function of this process.

(b) Show that $|R_{XX}(\tau)| \le R_{XX}(0)$.

OR

7 (a) Show that the autocorrelation function of a stationary random process is an even function of τ.

(b) Give the classification of random processes.

UNIT – IV

8 (a) A stationary random process has a two-sided spectral density given by:

$$S_{XX}(f) = \begin{cases} 10; & a < |f| < b \\ 0; & else\ where \end{cases}$$

Find the mean-square value of the process if $a = 4$ and $b = 5$.

(b) List the properties of power spectral density function.

OR

9 (a) For two jointly stationary random processes, the cross-correlation function is $R_{XY}(\tau) = 2e^{-2\tau}u(\tau)$. Find the two cross-spectral density function.

(b) List the properties of cross power spectral density function.

UNIT – V

10 Show that $R_{YY}(\tau) = R_{XX}(\tau) * h(\tau) * h(-\tau)$.

OR

11 Write short notes on the following:

(a) Bandpass random process.

(b) Band-limited random process.

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