

MATHEMATICS - I

(Common to all branches)

Time: 3 hours

Max. Marks: 70

Part – A
(Compulsory Question)

1 Answer the following: (10 X 02 = 20 M)

- (a) Solve $\frac{d^2y}{dx^2} + 1.5\frac{dy}{dx} + 0.5y = 0$.
- (b) Solve $(e^y + 1) \cos x dx + e^y \sin x dy = 0$.
- (c) Find Taylor's series expansion for $\tan^{-1}\left(\frac{y}{x}\right)$ about $(1, 1)$.
- (d) Find the radius of the curvature at the origin for the curve $2x^4 + 3y^4 + 4x^2y + xy - y^2 + 2x = 0$.
- (e) Find the asymptote of $y = \frac{x^2 + 2x - 1}{x}$.
- (f) Evaluate $\int_0^1 \int_0^x e^{x+y} dy dx$.
- (g) Find $L\{\cos^2 t\}$.
- (h) Find $L^{-1}\left\{\frac{e^{-3s}}{s+2}\right\}$.
- (i) Show that $\nabla \cdot (r^n \bar{r}) = (n+3)r^n$.
- (j) State Stokes theorem.

Part – B

Answer all five units (5 X 10 = 50 M)

UNIT - I

- 2 A mass m suspended from one end of a spring is subjected to a periodic force $f = f_0 \sin at$ in the direction of its length. The force f is measured positive vertically downwards and at time $t = 0$, m is at rest. If the spring constant is K , prove that the displacement of m at time t is given by $x = \frac{f_0}{m(p^2 - a^2)} \left(\sin at - \frac{a}{p} \sin pt \right)$ where $p^2 = K/m$. Neglect the damping effects.

OR

- 3 Solve $(x^2 D^2 + xD + 1)y = \log x \sin(\log x)$.

UNIT - II

- 4 Discuss the maxima and minima of $f(x, y) = \sin x \sin y \sin(x + y)$.

OR

- 5 Prove that the evolute of the cycloid $x = a(t - \sin t), y = a(1 - \cos t)$ is another cycloid.

UNIT - III

- 6 Find the length of the arc of the parabola $y^2 = 4ax$ cut off by the straight line $y = x$.

OR

- 7 Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z dz dx dy$.

UNIT - IV

- 8 Using convolution theorem solve the IVP:

$$y''(t) + 3y'(t) + 2y(t) = e^{-t}, \quad y(0) = 0, \quad y'(0) = -1$$

OR

- 9 Find $L^{-1}\left\{\frac{s(a^2 - b^2)}{s^4(a^2 + b^2)s^2 + a^2b^2}\right\}$.

UNIT - V

- 10 For a solenoidal vector \bar{f} , prove that $\nabla \times (\nabla \times (\nabla \times (\nabla \times \bar{f}))) = \nabla^4 \bar{f}$.

OR

- 11 Evaluate $\int_C [(2xy^3 - y^2 \cos x) dx + (1 - 2ys \sin x + 3x^2y^2) dy]$ where C is the arc of the parabola $2x = \pi y^2$ from $(0, 0)$ to $\left(\frac{\pi}{2}, 1\right)$.