Code: 13A54302 **R13**

B.Tech II Year I Semester (R13) Supplementary Examinations June 2016

MATHEMATICS – III

(Common to EEE, ECE and EIE)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) Evaluate $\int_0^1 x^4 (1-x)^2 \, dx$

(b) Compute $\beta(\frac{9}{2},\frac{7}{2})$

(c) Use Rodrigues formula, show that $P_0(x) = 1$.

(d) Prove that $x^2 = \frac{1}{3}P_0(x) + \frac{2}{3}P_2(x)$

(e) If $f(z)$ is an analytic function with constant modulus, show that $f(z)$ is constant.

(f) Define Bilinear transformation and write cross ratio of four points.

(g) Evaluate $\int_0^{2+i} z^2 \, dz$, along the line y = x/2.

(h) Write the statement of Cauchy's integral formula.

(i) Define isolated singular point of an analytic function.

(j) Evaluate $\int_C \frac{z-3}{z^2+2z+5} \, dz$, where C is circle $|z| = 1$.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

2 (a) Show that $\int_a^b (x-a)^m (b-x)^n \, dx = (b-a)^{m+n+1} \beta(m+1,n+1)$

(b) Show that $\int_0^\infty x^m e^{-x^n} \, dx = \frac{\Gamma(m)}{n^m}$, (m , n > 0).

OR

3 Obtain the series solution of the equation $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$.

UNIT – II

4 (a) Show that $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}(\frac{\sin x}{x} - \cos x)$

(b) Show that $x^4 = \frac{1}{35}[8P_4(x) + 20P_2(x) + 7P_0(x)]$

OR

5 (a) Prove that $J_{-\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}}(\frac{3}{x}\sin x + \frac{3-x^2}{x^2}\cos x)$

(b) Use Rodrigues formula, show that: (i) $P_1(x) = x$. (ii) $P_2(x) = \frac{3x^2-1}{2}$.

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UNIT – III

6 (a) If f(z) is an analytic function of z prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log|f(z)|=0$

(b) Find the analytic function f(z) = u + i v if u + v = $\frac{\sin 2x}{(\cosh 2y-\cos 2x)}$

OR

7 (a) Find the analytic function whose real part $u = \frac{\sin 2x}{(\cosh 2y-\cos 2x)}$.

(b) Find the bilinear transformation which maps the points (∞, i, 0) into the points (-1, -1, 1) in w- plane.

UNIT – IV

8 (a) Evaluate $\int_C \left(y-x-3x^2 i\right)dz$, where C is the straight line from Z = 0 to Z = 1+ i.

(b) Find Taylor's expansion of $f(z) = \frac{2z^3+1}{z^2+z}$ about the point z = i.

OR

9 (a) Evaluate:

$$\int_C \frac{\cos z - \sin z}{(z+i)^3}\, dz \ with\ C : |z| = 2 \ \text{using Cauchy's integral formula.}$$

(b) Expand $\frac{\tan z}{z}$ by Laurent's series and then find nature of singularity.

UNIT – V

10 (a) Determine the poles of the function $\frac{z^2+1}{z^2-2z}$ and the residue at each pole.

(b) Evaluate $\int_{-\infty}^{\infty}\frac{1}{x^4+1}\,dx$.

OR

11 Apply the calculus of residues, to prove that:

$$\int_{-\infty}^{\infty}\frac{\cos x}{(x^2+a^2)(x^2+b^2)}\,dx = \frac{\pi}{a^2-b^2}\left(\frac{e^{-b}}{b}-\frac{e^{-a}}{a}\right)\ (a>b>0)$$

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