Code: 13A54302

R13

B.Tech II Year I Semester (R13) Supplementary Examinations November/December 2016

MATHEMATICS – III

(Common to EEE, ECE and EIE)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) Express the integral $\int_0^\infty e^{-x^2}\,dx$ in terms of gamma function.

(b) Evaluate $\int_0^{\pi/2} \sqrt{\cot\theta}\,d\theta$

(c) Find the general solution of $y'' + \frac{y'}{x} + \left(1 - \frac{1}{9x^2}\right)y = 0$.

(d) Show that $P_1(x) = x$.

(e) Determine whether the function $u = x^2 - y^2$ is harmonic.

(f) Find the fixed points of the transformation $w = \frac{z-1}{z+1}$.

(g) Evaluate $\int_C \frac{(z^2 - z + 1)}{(z-1)(z-3)}dz$, where $C: |z| = \frac{1}{2}$.

(h) Find the nature and location of singularity of $f(z) = \frac{z - \sin z}{z^2}$.

(i) Find the residue of $f(z) = \frac{e^{-z}}{z^2}$ at $z = 0$.

(j) State residue theorem.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

2 (a) Show that $\int_0^1 x^m (\log x)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$, where n is a positive integer and $m > -1$.

(b) Obtain the series solution of the equation $\frac{d^2y}{dx^2} - y = 0$.

OR

3 (a) Obtain the relationship between beta and gamma functions.

(b) Solve the equation $y'' + xy = 0$ in series.

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UNIT – II

4 (a) Prove that $e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} t^n J_n(x)$.

(b) Show that $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$.

OR

5 (a) Show that $J_{-\frac{1}{2}}(x) = \sqrt{\left(\frac{2}{\pi x}\right)}\cos x$.

(b) Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = 0$, for $m \neq n$.

UNIT – III

6 (a) Find the analytic function, whose real part is $\frac{\sin 2x}{\cos h2y - \cos 2x}$.

(b) Show that the bilinear transformation $w = \frac{2z+3}{z-4}$ maps the circle $x^2 + y^2 - 4x = 0$ into the line $4u + 3 = 0$.

OR

7 (a) If $f(z)$ is an analytic function with constant modulus then show that $f(Z)$ is constant.

(b) Find the bilinear transformation which maps the points $1, i, -1$ of the z-plane onto $2, i, -2$ of the w-plane respectively.

UNIT – IV

8 (a) Find the value of $\int_{0}^{1+i}(x - y + ix^2)dz$ along real axis from $z = 0$ to $z = 1$ and then along a line parallel to the imaginary axis from $z = 1$ to $z = 1 + i$.

(b) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region : (i) $|z| < 1$ and (ii) $1 < |z| < 2$.

OR

9 (a) Using Cauchy's integral formula, evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)}dz$, where $C:|z| = 3$.

(b) Find the Laurent's expansion of $f(z) = \frac{7z+2}{(z+1)z(z-2)}$ in the region $1 < z+1 < 3$.

UNIT – V

10 (a) Evaluate $\int_C \frac{2z+1}{(2z-1)^2}dz$, where $C:|z| = 1$.

(b) Apply calculus of residues to prove that $\int_{0}^{2\pi} \frac{1}{17 - 8\cos\theta}d\theta = \frac{2\pi}{15}$.

OR

11 (a) Evaluate $\int_C \frac{\sin^2 z}{\left(z - \frac{\pi}{6}\right)^2}dz$, where $C:|z| = 2$.

(b) Prove that $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)}dx = \frac{\pi}{3}$ by calculus of residues.

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