

**MATHEMATICS – I**  
(Common to all branches)

Time: 3 hours

Max. Marks: 70

**PART – A**  
(Compulsory Question)

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- 1 Answer the following: (10 X 02 = 20 Marks)
- Solve  $[\cos x \tan y + \cos(x+y)]dx + [\sin x \sec^2 y + \cos(x+y)]dy = 0$ .
  - Solve  $(x+1)\frac{dy}{dx} - y = e^x(x+1)^2$ .
  - Find the particular integral of  $(D^2 + 4D + 4)y = \frac{e^{-2x}}{x^2}$ .
  - Solve  $(x^2D^2 - xD + 1)y = 0$ .
  - Find the radius of curvature for the curve  $y = 4\sin x - \sin 2x$  at  $x = 90^\circ$ .
  - If  $u = \frac{y^2}{x}$ ,  $v = \frac{x^2}{y}$  find  $\frac{\partial(u,v)}{\partial(x,y)} = ?$
  - Evaluate  $\int_0^a \int_0^{\sqrt{ay}} xy \, dx \, dy$ .
  - Evaluate  $\int_0^\pi \int_0^{a\cos\theta} r \sin\theta \, dr \, d\theta$ .
  - Find the unit vector normal to the surface  $x^2 - y^2 + z = 2$  at the point  $(1, -1, 2)$ .
  - Show that  $\vec{F} = (y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2x^2z - y + 2z)\vec{k}$  is irrotational.

**PART – B**

(Answer all five units, 5 X 10 = 50 Marks)

**UNIT – I**

- Solve  $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2 + 3$ .
  - Prove that the system of confocal and coaxial parabolas  $y^2 = 4a(x+a)$  is self orthogonal.

OR

- Solve  $(D^2 + 4D + 3)y = e^x \sin x$ .
  - Solve  $\frac{dy}{dx} + \frac{x}{1-x^2}y = x\sqrt{y}$ .

**UNIT – II**

- Solve  $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$  by using method of variation of parameters.

OR

- Solve  $(x+2)^2 \frac{d^2y}{dx^2} - (x+2) \frac{dy}{dx} + y = 3x + 4$ .

**UNIT – III**

- Expand  $\sin(xy)$  in powers of  $(x-1)$  and  $(y-\pi/2)$  upto second degree terms.
  - Discuss the maximum and minimum of  $f(x,y) = x^3 + y^3 - 12x - 3y + 20$ .

OR

- If  $x = r \sin\theta \cos\theta$ ,  $y = r \sin\theta \sin\phi$  and  $z = r \cos\theta$  find  $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$ .
  - Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.

**UNIT – IV**

- Evaluate  $\iint_R (x^2 + y^2) \, dx \, dy$ , where R is the square  $0 \leq x \leq a$ ,  $0 \leq y \leq a$ .
  - Transform the integral into polar-co-ordinates and hence evaluate  $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2 + y^2} \, dy \, dx$ .

OR

- Change the order of integration in  $\int_0^a \int_x^a (x^2 + y^2) \, dy \, dx$  and then evaluate.
  - Find the area included between the curves  $y^2 = 4x$  and  $x^2 = 4y$ .

**UNIT – V**

- If  $\vec{F} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$ , evaluate  $\int_C \vec{F} \cdot d\vec{r}$  where C is the straight line joining  $(0,0,0)$  to  $(1,1,1)$ .
  - Using Stokes theorem, evaluate  $\int_C \vec{F} \cdot d\vec{r}$  for the function  $\vec{F} = x^2\vec{i} + xy\vec{j}$  in XOY-plane bounded by  $x=0$ ,  $y=0$ ,  $x=a$ ,  $y=a$ .

- Verify Divergence theorem for  $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$  taken over the rectangular parallelepiped  $x=0$ ,  $x=1$ ,  $y=0$ ,  $y=2$ ,  $z=0$  and  $z=3$ .

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