

MATHEMATICS – I

(Common to CE, EEE, CSE, ECE, ME, EIE and IT)

Time: 3 hours

Max. Marks: 70

PART – A

(Compulsory Question)

1 Answer the following: (10 X 02 = 20 Marks)

(a) If $x = r \cos \theta$, $y = r \sin \theta$ find $\frac{\partial(x, y)}{\partial(r, \theta)}$.

(b) Find Particular Integral of $(D^2 + 1)y = \cosh 2x$

(c) Find the orthogonal trajectories of the family of curve $ay^2 = x^3$

(d) Solve $y'' + 6y' + 9y = 0$, $y(0) = -4$ and $y'(0) = 14$

(e) Solve $\frac{dy}{dx} + y \tan x = \cos^3 x$

(f) State Newton's law of cooling.

(g) State Stokes theorem.

(h) In what direction from (3,1,-2), direction derivative of $f = x^2y^2z^4$ is maximum. Find the Maximum value.

(i) Evaluate $\int_1^a \int_1^b \frac{dydx}{xy}$

(j) Find the unit normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point (1, 2, -1).**PART – B**

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

2 (a) Solve $(1 + y^2) + (x - e^{\tan^{-1}y}) \frac{dy}{dx} = 0$

(b) The number N of bacteria in culture grew at a rate proportional to N. The value of N was initially 100 and increases to 332 in one hour. What was value of N after 1 ½ hours.

OR

3 (a) Solve $(D^2 - 1)y = xe^x \sin x$

(b) Prove that the system of parabolas $y^2 = 4a(x + a)$ is self orthogonal**UNIT – II**

4 Solve $(D^2 + a^2)y = \tan ax$ by method of variation of parameter.

OR

5 Solve $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} - 5y = \sin(\log x)$

Contd. in page 2

UNIT – III

- 6 (a) Verify whether the following functions are functionally dependence, if so, find the relation between them $u = \frac{x+y}{1-xy}$, $v = \tan^{-1} x + \tan^{-1} y$.
- (b) Examine for Maxima and Minima of $\sin x + \sin y + \sin(x+y)$

OR

- 7 Find a point at the plane $3x + 2y + z - 12 = 0$ which is nearest to the origin.

UNIT – IV

- 8 Evaluate the following integral by changing the order of integration $\int_0^1 \int_{x^2}^{2-x} xy dx dy$

OR

- 9 (a) Show that the double integration, the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$.

- (b) Evaluate the $\int_0^1 \int_y^{1-x} \int_0^{1-x} x dz dx dy$

UNIT – V

- 10 (a) Prove that $\text{div}(\text{grad } r^m) = m(m+1)r^{m-2}$
- (b) Find the directional derivative of $f = xy + yz + zx$ in the direction of vector $i + 2j + 2k$ at the point $(1, 2, 0)$.

OR

- 11 Verify Green's theorem in the plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the region by $y = \sqrt{x}$ and $y = x^2$.
