B.Tech I Year I Semester (R15) Regular \& Supplementary Examinations December 2016 MATHEMATICS - I
(Common to CE, EEE, CSE, ECE, ME, EIE and IT)
Time: 3 hours
Max. Marks: 70

## PART - A

(Compulsory Question)
1 Answer the following: ( $10 \times 02=20$ Marks $)$
(a) Find the orthogonal trajectories of the family of parabolas through the origin and foci on the $y$ - axis.
(b) Find the complementary function $\left(D^{3}+2 D\right) y=e^{2 x}+\cos (3 x+7)$.
(c) $x^{2} \frac{d^{2} y}{d x^{2}}+3 x \frac{d y}{d x}=0$ has the general solution $\qquad$
(d) Find P.I $\left(\theta^{2}-4 \theta+1\right)^{-1} \sin \mathrm{z}$.
(e) If $u=e^{x+y}, v=e^{-x+y}$, then find $J$.
(f) Find the radius of curvature at any point of the cardioids $s=4 \operatorname{asin} \frac{\psi}{3}$.
(g) $\int_{\mathrm{D}} \int\left(\mathrm{x}^{2}+\mathrm{y}^{2}\right) \mathrm{dxdy}=$ $\qquad$ D: $y=x, y^{2}=x$.
(h) Evaluate $\int_{0}^{1} \mathrm{dx} \int_{1}^{2} \mathrm{dy} \int_{1}^{3} \mathrm{xyzdz}$.
(i) $\nabla \times(\nabla \times \overline{\mathrm{A}})$ is $\qquad$
(j) Evaluate $\int_{c} y^{2} d x-2 x^{2} d y$ along the parabola $y=x^{2}$ from $(0,0)$ to $(2,4)$.

PART - B
(Answer all five units, $5 \times 10=50$ Marks)

## UNIT - I

2
Solve: $x(x-1) \frac{d y}{d x}-y=x^{2}(x-1)^{3}$.
Solve: $\left(D^{3}+2 D^{2}-3 D\right) y=x e^{3 x}$.

## OR

## UNIT - II

Solve: $\left(D^{2}+a^{2}\right) y=\tan a x$ by the method of variation of parameters.
OR
The deflection $y$ of a strut of length $l$ with one end built-in and other end subjected to the end thrust $P$, satisfies $\frac{d^{2} y}{d x^{2}}+a^{2} y=\frac{a^{2} R}{P}(1-x)$. Find the deflection $y$ of the strut at $a$ distance $x$ from the built-in end.

## UNIT - III

6 (a) If $u=\sin ^{-1}\left(\frac{x^{2} y^{2}}{x+y}\right)$ then show that $x u_{x}+y u_{y}=3 \tan u$.
(b) If $u=x+y+z, u v=y+z, u v w=z$, then prove $\frac{\partial(x, y, z)}{\partial(u, v, w)}=u^{2} v$.

## OR

7 (a) Find the points on the surface $z^{2}=x y+1$ nearest to the origin.
(b) Find the radius of curvature at $(3,3)$ on the curve $\mathrm{x}^{3}+\mathrm{xy}^{2}-6 \mathrm{y}^{2}=0$.

Contd. in page 2

## UNIT - IV

8 Evaluate $\int_{0}^{1} \int_{0}^{\sqrt{1-\mathrm{x}^{2}}} \mathrm{y}^{2} \mathrm{dxdy}$ by changing the order of integration.
OR
9
Evaluate $\iiint x^{2} z d x d y d z$ taken through the positive octant of the sphere: $x^{2}+y^{2}+z^{2}=a^{2}$.

## UNIT - V

10 (a) Find the directional derivative of $f=x y+y z+z x$ in the direction of vector $\bar{i}+2 \bar{j}+2 \bar{k}$ at the point $(1,2,0)$.
(b) Find curl $\bar{f}$ where $\bar{f}=\operatorname{grad}\left(x^{3}+y^{3}+z^{3}-3 x y z\right)$.

OR
11 Evaluate by Green's theorem $\oint_{c}(y-\sin x) d x+\cos x d y$ where $C$ is triangle enclosed the lines $y=0, x=\frac{\pi}{2}, \pi y=2 x$.

