

MATHEMATICS – II

(Common to all)

Time: 3 hours

Max. Marks: 70

PART – A

(Compulsory Question)

- 1 Answer the following: (10 X 02 = 20 Marks)
- Find $L [t^2 \cdot e^t \cdot \cos 4t]$
 - Define unit step function Laplace transform.
 - If $f(x) = x^4$ in $(-1, 1)$ then find the Fourier coefficient of b_n .
 - What is Fourier even function $(-\pi, \pi)$?
 - Write the Fourier sine transform of $f(t)$.
 - Find the value of $Z(a^n \cos nt)$
 - Find the general solution of $u_{xx} = xy$.
 - Find the Z-transform of the sequence $\{x(n)\}$ where $x(n)$ is $n \cdot 2^n$
 - Find $z^{-1} \left(\frac{1}{z-3} \right)$.
 - What do you mean by steady state and transient state?

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

- 2 (a) Apply convolution theorem for $L^{-1} \left(\frac{1}{s^3(s^2+1)} \right)$.
- (b) Evaluate $L \left(e^{-t} \int_0^t \frac{\sin t}{t} dt \right)$.

OR

- 3 Solve $\frac{d^2x}{dt^2} + 9x = \cos 2t$, if $x(0) = 1$, $x \left(\frac{\pi}{2} \right) = -1$.

UNIT – II

- 4 Find the Fourier series expansion of $f(x) = 2x - x^2$ in $(0, 3)$ and hence deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots \infty = \frac{\pi}{12}$.

OR

- 5 Obtain half range cosine series for $f(x) = \begin{cases} kx & , 0 \leq x \leq \frac{l}{2} \\ k(l-x), & \frac{l}{2} \leq x \leq l \end{cases}$. Deduce the sum of the series $\frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} + \dots \infty$.

UNIT – III

- 6 (a) Find the Fourier sine transform of $e^{-|x|}$.
- (b) Write the conditions of Parseval's identity for Fourier transforms.

OR

- 7 Verify convolution theorem for $f(x) = g(x) = e^{-x^2}$.

UNIT – IV

- 8 (a) Form the partial differential equation $z = f \left(\frac{xy}{2} \right)$ by eliminating the arbitrary function.
- (b) Use the method of separation of variables, solve $\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$ where $u(x, 0) = 8e^{-3y}$.

OR

- 9 Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions $u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, a) = \sin \left(\frac{n\pi x}{l} \right)$.

UNIT – V

- 10 (a) Find the z-transformation of $\sin n\theta$.
- (b) If $U(z) = \frac{2z^2 + 5z + 14}{(z-1)^4}$, evaluate U_2, U_3 using initial value theorem.

OR

- 11 Solve the differential equation $u_{n+2} - 2u_{n+1} + u_n = 3n + 5$ using z-transforms.
