

**7002****BOARD DIPLOMA EXAMINATION, (C-20)****OCTOBER/NOVEMBER—2023****FIRST YEAR (COMMON) EXAMINATION****ENGINEERING MATHEMATICS—I**

Time : 3 Hours]

[Total Marks : 80

PART—A

3×10=30

Instructions : (1) Answer **all** questions.(2) Each question carries **three** marks.

1. If $A = \{1,2,3,4\}$ and $f : A \rightarrow \mathbb{R}$ defined by $f(x) = x^2 + x + 1$, then find the range of f .
2. Resolve $\frac{1}{(x+1)(x+2)}$ into partial fractions.
3. If $A = \begin{pmatrix} -1 & 4 \\ 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$, then find $2A - B$.
4. Show that $\frac{\cos 36^\circ + \sin 36^\circ}{\cos 36^\circ - \sin 36^\circ} = \cot 9^\circ$
5. Prove that $\cos(45^\circ + \theta)\cos(45^\circ - \theta) = \frac{1}{2} \cos 2\theta$
6. Find the modulus of the complex number $\frac{2+i}{1-i}$.
7. Find the equation of the straight line passing through the points $(1, -1)$ and $(-2, 3)$.

8. Evaluate $\lim_{x \rightarrow 0} \frac{\sin 13x}{\sin 39x}$
9. Find the derivative of $e^x \sin x$ w.r.t. x .
10. If $x = a \sin \theta$ and $y = a \cos \theta$, then find $\frac{dy}{dx}$.

PART—B

8×5=40

- Instructions :** (1) Answer **all** questions.
(2) Each question carries **eight** marks.

11. (a) Solve the system of linear equations $2x - y + 3z = 9$, $x + y + z = 6$ and $x - y + z = 2$ using Cramer's rule.

(OR)

(b) If $A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$, then find A^{-1} .

12. (a) If $\cos x + \cos y = \frac{1}{3}$ and $\sin x + \sin y = \frac{1}{4}$, then find the values of $\cos(x + y)$ and $\sin(x + y)$.

(OR)

(b) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, then prove that $x + y + z = xyz$.

13. (a) Solve $\cos 8\theta + \cos 2\theta = \cos 5\theta$

(OR)

(b) In any $\triangle ABC$, if $\angle B = 60^\circ$, then show that $\frac{c}{a+b} + \frac{a}{b+c} = 1$

14. (a) Find the equation of the circle passing through the points $(0, 0)$, $(-2, 0)$ and $(1, 3)$.

(OR)

(b) Find the equation of the rectangular hyperbola with focus (1, 2) and equation of the directrix is $2x + y - 1 = 0$.

15. (a) If $x^y = e^{x-y}$, then show that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$.

(OR)

(b) Find the derivative of $\tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$ w.r.t. $\cot^{-1} x$.

PART—C

10×1=10

- Instructions :**
- (1) Answer the following question.
 - (2) The question carries **ten** marks.
 - (3) Answer should be comprehensive and the criterion for valuation is the content but not the length of the answer.

16. Find the equations of tangent, normal, lengths of tangent, normal, sub-tangent and sub-normal to the curve $y = x^2 - 6x + 8$ at the point (1, 3).

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