# JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD <br> B.Tech I Year Examinations, May/June - 2017 <br> MATHEMATICAL METHODS <br> (Common to EEE, ECE, CSE, EIE, IT, ETM) 

Time: 3 hours
Max. Marks: 75
Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have $\mathrm{a}, \mathrm{b}, \mathrm{c}$ as sub questions.

## PART- A ( 25 Marks)

1.a) Define any 4 difference operators.
b) Prove that $\frac{\Delta}{\nabla}-\frac{\nabla}{\Delta}=\Delta+\nabla$
c) Write the iterative formula for finding the approximate solution of the initial value problem $\frac{d y}{d x}=f(x, y) ; y\left(x_{0}\right)=y_{0}$.
d) Find the positive square root of 12 up to 4 decimal places.
e) If $x=\sum_{n=1}^{\infty} \frac{2 b_{n}}{\pi} \sin n x, 0 \leq x \leq \pi$, find $b_{n}$.
f) Does the Fourier series expansion of $f(x)=1,0<x<4, f(x+4)=f(x)$ exist? If so, find the constant term, coefficients of $\cos \pi x$ and $\sin \pi x$.
g) Solve $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=1$
h) Write all possible solutions of the equation $\frac{\partial u}{\partial t}=c^{2} \frac{\partial^{2} u}{\partial x^{2}}$
i) State Gauss divergence theorem.
j) If $\nabla \times \bar{A}=\overline{0}, \nabla \times \bar{B}=\overline{0}$ then find the value of $\operatorname{Div}(\bar{A} \times \bar{B})$.

## PART-B (50 Marks)

2.a) Using Gauss's backward interpolation formula find the population for the year 1936 given that,

| Year x | 1901 | 1911 | 1921 | 1931 | 1941 | 1951 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Population y | 12 | 15 | 20 | 27 | 39 | 52 |

b) Prove that $\left(E^{\frac{1}{2}}+E^{-\frac{1}{2}}\right)(1+\Delta)^{\frac{1}{2}}=2+\Delta$

## OR

3.a) Using the following table, find $\mathrm{f}(2.75)$ using Forward difference formula.

| x | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{Y}=\mathrm{f}(\mathrm{x})$ | 21.145 | 22.043 | 20.225 | 18.644 | 17.262 | 16.047 |

b) Using the following table fit a curve of the form $y=a x^{b}$ using method of least squares.

4.a) Find a root of the equation $x^{3}-9 x+1=0$ correct to 4 decimal places by bisection method.
b) Solve the following system of equations by using Gauss -Seidal iterative method (give the solution correct to 3 decimal places) $8 x-3 y+2 z=20 ; 4 x+11 y-z=33 ; \quad 6 x+3 y+12 z=35$.

## OR

5.a) By applying $4^{\text {th }}$ order Runge-Kutta method obtain the values of $y$ at $x=0.1$ and at 0.2 for the differential equation $\frac{d y}{d x}=-y$, given that $y(0)=1$.
b) Apply Simpson's rule to find the value of $\int_{0}^{2} \frac{1}{1+x^{3}} d x$ by taking 10 points in [0, 2]. [5+5]
6.a) Obtain the Fourier series of $f(x)=f(x+2 \pi)$ and $f(x)=\frac{\pi-x^{2}}{4}, 0<x<2 \pi$.
b) Find the half range cosine series of $f(x)=x(2-x)$ in $0 \leq x \leq 2$.

## OR

7.a) Find the half range cosine series for the function $f(x)=\left\{\begin{array}{l}x^{2}, 0 \leq x<1 \\ 1,1 \leq x \leq 2\end{array}\right.$
b) Is the function defined as $f(x)=\left\{\begin{array}{l}x+\pi, 0 \leq x \leq \pi \\ x-\pi,-\pi \leq x \leq 0\end{array}\right.$ even or odd? If $f(x+2 \pi)=f(x)$, find its Fourier series expansion.
8.a) Using the method of separation of variables, solve $\frac{\partial u}{\partial x}=2 \frac{\partial u}{\partial t}+u$, where $u(x, 0)=6 \mathrm{e}^{-3 \mathrm{x}}$.
b) Solve $z=p^{2} x+q^{2} y$, using Charpit's method.

## OR

9.a) An insulated rod of length $l$ has its ends A and B maintained at $0^{\circ} \mathrm{C}$ and $100^{\circ} \mathrm{C}$ respectively until steady state conditions prevail. If $B$ is suddenly reduced to $0^{0} \mathrm{C}$ and maintained at $0^{\circ} \mathrm{C}$, find the temperature at a distance x from A at time t .
b) Solve $\left(x^{2}-y z\right) p+\left(y^{2}-x z\right) q=\left(z^{2}-y x\right)$
10.a) Find the directional derivative of $\phi=x^{2} y z+4 x z^{2}$ at the point $(1,-2,-1)$ in the direction of the vector $2 \bar{i}-\bar{j}-2 \bar{k}$
b) Prove that $\nabla \times \nabla \times \bar{F}=\nabla(\nabla \cdot \bar{F})-\nabla^{2} \bar{F}$

OR
11. Verify Stoke's Theorem for $\bar{A}=(2 x-y) \bar{i}-y z^{2} \bar{j}-y^{2} z \bar{k}$ over upper half of the surface of the sphere of unit radius.

