R15

Code No: 123AN

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, November/December - 2016

PROBABILITY AND STATISTICS

(Common to ME, CSE, IT, MCT, AME, MIE, MSNT)

Time: 3 Hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

1.a) What is the expected number of heads appearing when a fair coin is tossed three times? [2]

b) Prove that the total area under the normal curve is unity. [3]

c) Prove that correlation coefficient is the geometric mean of the two regression coefficients. [2]

d) Define covariance of two random variables. When are two random variables uncorrelated. [3]

e) Define Type-I and Type-II errors. [2]

f) A sample of size 10 drawn from a normal population has a mean 31 and variance 2.25. Is it reasonable to assume that the mean of the population is 30? Use 1% LOS. [3]

g) Define transient state and steady state in a queue model. [2]

h) Explain the operating characteristics of a queueing system. [3]

i) Write down the Chapman-Kolmogorov equations. [2]

j) If the transition probability matrix of a Markov chain is $\begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$, find the steady state distribution. [3]

PART-B

(50 marks)

2.a) A random variable X is defined as the sum on the faces when a pair of dice is thrown. Find the probability mass function of X and the expected value of X.

b) Explain Binomial distribution. Derive its moment generating function and hence find its mean and variance. [5+5]

OR

3.a) Define mathematical expectation. Prove the multiplication theorem of expectation.

b) Explain normal distribution. If the mean height of sorghum varieties to be 68.22 inches with a variance of 10.8 inches, how many varieties in a field of 100 varieties, would you expect to have 6 feet tall? [5+5]

4.a) Obtain the rank correlation coefficient for the following data:

X	68	64	75	50	64	80	75	40	55	64
Y	62	58	68	45	81	60	68	48	50	70

b) The joint distribution of X and Y is given by $f(x,y) = 4xye^{-(x^2+y^2)}$; $x \geq 0,\ y \geq 0$. Find the marginal density functions of X and Y and test whether X and Y are independent. [5+5]

OR

5.a) The following data pertain to the marks in subjects A and B in a certain examination: Mean marks in A=39.5; Mean marks in B=47.5; Standard deviation of marks in A=10.8; Standard deviation of marks in B=16.8. Coefficient of correlation between marks in A and marks in B=0.42. Compute the two lines of regression and explain why there are two regression equations. Give the estimate of marks in B for candidates who secured 50 marks in A.

b) Two independent variables are defined as

$f(x) = \begin{cases} 4ax & 0 \leq x \leq r \\ 0 & otherwise \end{cases}$, $f(y) = \begin{cases} 4by & 0 \leq y \leq s \\ 0 & otherwise \end{cases}$. If $U = X + Y$ and $V = X - Y$ then show that $Cov(U,V) = \frac{b-a}{b+a}$. [5+5]

6.a) Fit a Poisson distribution to the following data and test for the goodness of fit:

X:	0	1	2	3
Frequency:	24	15	6	5

b) Two independent samples of sizes 8 and 7 items respectively had the following values.

Sample I	11	11	13	11	15	9	12	14
Sample II	9	11	10	13	9	8	10	--

Is the difference between the means of the sample significant? Test at 5% LOS. [5+5]

OR

7.a) Explain the concepts of confidence intervals and the standards error of an estimate. The mean and variance of random sample of 64 observations were computed as 160 and 100, respectively. Compute the 95% confidence limits for population mean.

b) Two random samples are drawn from two populations and the following results are obtained:

Sample I	16	17	18	19	20	21	22	23	24
Sample II	12	17	18	22	27	23	32	--	--

Find the variances of the two samples and test whether the two populations have the same variance at 5% level of significance. [5+5]

8. Obtain the steady state solution of the system $(M/M/1):(\infty/FCFS)$. Find the probability that atleast one unit is present in the system and also find the expected queue length. [10]

OR

9. Define pure birth-death processes. Cars arrive at a pollution testing center according to Poisson distribution at an average rate of 15 cars per hour. The testing center can accommodate at maximum 15 cars. The service time per car is an exponential distribution with mean rate 10 per hour. (a) What is the probability that an arriving car does not have to wait for testing. (b) What is the expected waiting time until a car is left from the testing center. [5+5]

10.a) Define Markov process and Markov chain. Prove that the Poisson process is a Markov process.

b) Describe stationary and non-stationary random process. [5+5]

OR

11.a) Define stochastic process and stochastic matrix. Give examples. When is a stochastic matrix said to be regular?

b) Prove that the matrix $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ is the transition probability matrix of an irreducible Markov chain. [5+5]

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