# JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD <br> B.Tech II Year I Semester Examinations, December - 2019 <br> PROBABILITY AND STATISTICS <br> (Common to ME, CSE, IT, MCT, AME, MIE) 

## Time: 3 Hours

Max. Marks: 75
Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have $\mathrm{a}, \mathrm{b}, \mathrm{c}$ as sub questions.

## PART- A

(25 Marks)
1.a) The mean and the variance of a Binomial distribution are 6 and 4 , then find $n$.
b) A random variable X has probability density function
$f(x)=\left[\begin{array}{l}\frac{1}{b} e^{-x / b}, x>0, b>0 \\ 0 \text { elsewhere }\end{array}\right.$ Find the mean of that random variable.
c) The joint probability density function of two random variables is given by
$f(x, y)=\left\{\begin{array}{cc}\frac{x}{5}+K y, & 0<x<1,1<y<5 \\ 0, & \text { elsewhere }\end{array}\right.$. . Find the value of K.
d) If the two lines of regression are $\mathrm{y}=0.4 \mathrm{x}+.09$ and $\mathrm{x}=0.7 \mathrm{y}+1.0$, then find r . [3]
e) A sample of 100 items is taken from a population whose standard deviation is 5 Find the standard error of means.
f) A random sample of 100 apples were taken and out of which 15 were found to be bad. Find the maximum error of proportions.
g) Define mean arrival rate in queuing theory.
h) Workers come to a tool store room to enquire about special tools. The average time between two arrivals is 90 seconds and the arrivals are assumed to be in Poisson distribution. The average service time is 50 seconds. Determine Average queue length.
i) Define an Absorbing Markov Chain.
j) Find the values of $\mathrm{x}, \mathrm{y}, \mathrm{z}$, if $\left[\begin{array}{ccc}0 & x & 1 / 3 \\ 0 & 0 & y \\ 1 / 3 & 1 / 4 & z\end{array}\right]$ is a transition probability matrix.

PART- B
(50 Marks)
2.a) A discrete random variable has the following distribution. Find:
i) mean
ii) variance.

| x | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{P}(\mathrm{x})$ | $\frac{1}{9}$ | $\frac{2}{9}$ | $\frac{3}{9}$ | $\frac{2}{9}$ | $\frac{1}{9}$ |

b) Given that the mean heights of students in a class is 158 cms with standard deviation of 20 cms . Find how many students heights lie between 150 cms and 170 cms if there are 100 students in the class.

## OR

3.a) Average number of accidents on any day on a national high way is 1.8. Determine the probability that the number of accidents are i) At least one ii) At the most one.
b) The mean inside diameter of a sample of 200 washers produced by a Machines is 0.500 cms with standard deviation 0.005 cms . The purpose of which these washers are intended a maximum tolerance in the diameter. 0.495 to 0.505 cms . otherwise the washers are considered defective. Determine the percentage of defective washers produced by the machine, assuming the diameters are normally distributed.
4. The marks obtained by 10 students in Mathematics and Statistics are given below. Find the Coefficient of correlation between the two subjects.
[10]

| Marks in Maths | 75 | 30 | 60 | 80 | 53 | 35 | 15 | 40 | 38 | 48 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Marks in Statistics | 85 | 45 | 54 | 91 | 38 | 63 | 35 | 43 | 45 | 44 |

OR
5. If $X$ and $Y$ are two random variables with joint probability density function

$$
f(x, y)=\left[\begin{array}{l}
\mathrm{Ke}^{-2 x-y}, \text { if } \mathrm{x}>0, \mathrm{y}>0 \\
0, \text { else } \text { where }
\end{array}\right.
$$

a) Find the value of $K$
b) Marginal probability density function for X
c) Marginal probability density function for $Y$

Find whether X and Y are independent.
6.a) A simple sample of 6400 Englishmen has a mean height of 67.85 inches and S.D. 2.56 inches, while a simple sample of 1,600 Australians has a mean height of 68.55 inches and a S.D of 2.52 inches. Do the data indicate that Australians are on the average, taller than Englishmen?
b) A sample of 400 items is taken from a population whose standard deviation is 10 . The mean of the sample is 40 . Test whether the sample has come from a population with mean 38.

## OR

7. Two independent samples of 8 and 7 times respectively has the following values.

| Sample I | 11 | 11 | 13 | 11 | 15 | 9 | 12 | 14 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sample II | 9 | 11 | 10 | 13 | 9 | 8 | 10 | - |

Is the difference between the means of sample significant?
8. Consider a self service store with one cashier. Assume Poisson arrivals and exponential service time. Suppose that 9 customers arrive on the average of every 5 minutes and the cashier can serve 10 in 5 minutes. Find:
a) The average number of customers queuing for service.
b) The probability of having more than 10 customers in the system
c) The probability that the customer has to queue for more than 2 minutes.
9. A bank has one drive counter. It is estimated that cars arrive according to Poisson distribution at the rate of 2 for every 5 minutes and that there is enough space to accommodate a line of 10 cars. Other arriving cars can wait outside this space if necessary. It takes 1.5 minutes on an average to serve a customer, but the service time actually varies according to an exponential distribution. Find
a) The proportion of the time the facility remains idle.
b) The expected number of customers waiting but currently not being served at a particular point of time.
c) The expected time a customer spends in the system
d) The probability that the waiting line will exceed the capacity of the space leading to the driver in counter.
10. The weather in a certain spot is classified as fair, cloudy,(without rain), or rainy. A fair day is followed by a fair day $60 \%$ of the time and by a cloudy day $25 \%$ of the time. A cloudy day is followed by a cloudy day $35 \%$ of the time and by a rainy day $25 \%$ of the time. A rainy day is followed by a cloudy day $40 \%$ of the time and by a rainy day $25 \%$ of the time. Initial probabilities are $0.3,0.3$ and 0.4 Find the probability that there will be rainy day after 3 days. What portion of the days is expected to be Fair, cloudy or rainy in long run?

## OR

11. If the transition probability matrix of market shares of three brands $\mathrm{A}, \mathrm{B}$ and C is $\left[\begin{array}{ccc}0.4 & 0.3 & 0.3 \\ 0.8 & 0.1 & 0.1 \\ 0.35 & 0.25 & 0.4\end{array}\right]$ and the initial market shares are $50 \%, 25 \%$ and $25 \%$. Find:
a) The market shares in second and third periods
b) The limiting probabilities.
