JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

## B.Tech II Year I Semester Examinations, April/May - 2018

MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE
(Common to CSE, IT)
Time: 3 Hours
Max. Marks: $\mathbf{7 5}$
Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have $\mathrm{a}, \mathrm{b}, \mathrm{c}$ as sub questions.

## PART- A

(25 Marks)
1.a) Find the negations of the following quantified statements:

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\begin{equation*}
\forall x, \exists y,[(p(x, y) \wedge q(x, y)) r(x, y)] \tag{2}
\end{equation*}
$$

b) Construct a truth table to show that $(\mathrm{p} \wedge \mathrm{q}) \rightarrow \mathrm{p}$ is a tautology.
c) Let $X=\{1,2,3,4,5,6\}$ and R be a relation defined as $(x, y) \in R$ if and only if $x-y$ is divisible by 3 . Find the elements of relation of $R$.
d) If ' $a$ ' is a generator of a cyclic group G, then show that $a^{-1}$ is also a generator of G. [3]
e) Find how many different words that can be formed with the letters in the word "MATHEMATICS"?
f) What is pigeon hole principle?
g) What is homogeneous recurrence relation?
h) Find the number of non-negative integer solutions of the equation $x_{1}+x_{2}+x_{3}=11$.[3]
i) What is chromatic numbers?
j) Define Euler's circuit and Give an example.

## PART-B

(50 Marks)
2.a) Verify the validity of the following arguments.
"Every living thing is a plant or an animal.
Logu's dog is alive and it is not a plant.
All animals have heart. Therefore Logu's dog has a heart."
b) Find the formulas in Disjunctive Normal Form equivalent to the following well formed formulas
$(\neg \mathrm{R}) \rightarrow(((\mathrm{P} \vee \mathrm{Q}) \rightarrow \mathrm{R}) \rightarrow \neg \mathrm{Q})$

## OR

3. Without using truth tables prove that $((\mathrm{P} \vee \mathrm{Q}) \neg(\neg \mathrm{P}(\neg \mathrm{Q} \vee \neg \mathrm{R}))) \vee(\neg \mathrm{P} \neg \mathrm{Q}) \vee(\neg \mathrm{Q} \neg \mathrm{R})$ is a tautology.
4.a) Let $A=\{a, b, c\}$ be a set and relation R on A is as $=\{(a, a),(a, b),(b, c),(c, c)\}$. Is R.
i) Reflexive ii) Symmetric iii)Transitive.
b) Prove that $f^{-1} \cdot g^{-1}=(g \cdot f)^{-1}$, where $f: Q \rightarrow Q$ such that $f(x)=2 x$ and $g: Q \rightarrow Q$ Such that $g(x)=x+2$ are two functions.

## OR

5.a) Prove that the intersection of any two subgroups of a group $G$ is again subgroup of $G$.
b) In a lattice $(\mathrm{L}, \leq, \wedge, \vee)$ state and prove the laws indempotent, commutative, association and absorption.
6.a) Find the number of integers between 1 and 250 that are divisible by any of the integers 2 , 3 and 6.
b) Find the coefficient of $x^{9} y^{3}$ in the expansion of $(2 x-3 y)^{12}$.

## OR

7.a) How many bit strings of length 10 contain:
i) At most four 1's ii) At least four 1's iii) Exactly four 1's.
b) There are 40 computer programmers for a job. 25 know Java, 28 know Oracle and 7 know neither language. Using principle of inclusion exclusion find how many know both languages.
8.a) Find a generating function for the recurrence relation $a_{n}-a_{n-1}+6 a_{n-2}=0$ for $n \geq 2$
b) Express Fibonacci sequence of numbers $1,1,2,3,5,8,13,21,34, \ldots$ in terms of general expression for the $r^{\text {th }}$ number $a_{r}$ and generating function.

## OR

9.a) Find the number of integer solutions of the equation
$\mathrm{x}_{1}+\mathrm{x}_{2}+\mathrm{x}_{3}+\mathrm{x}_{4}+\mathrm{x}_{5}=30$
Under the constraints $\mathrm{x}_{\mathrm{i}} \geq 0$ for all $\mathrm{i}=1,2,3,4,5$ and further $\mathrm{x}_{2}$ is even and $\mathrm{x}_{3}$ is odd.
b) Solve the recurrence relation $a_{n}-6 a_{n-1}+9 a_{n-2}=0$ for $n \geq 2$.
10.a) Show that a simple complete digraph with n nodes has the maximum number of edges $n(n-1)$. Assuming that there are no loops.
b) State and explain graph coloring problem. Give its applications.

OR
11.a) Find the minimum spanning tree by using kruskal's algorithm.

b) Write short notes on DFS and BFS.

