R16

Code No: 132AB

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech I Year II Semester Examinations, April - 2018
MATHEMATICS-II
(Common to EEE, ECE, CSE, EIE, IT, ETM)

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

1.a) Find $L\left\{t^2\ u(t-1)\right\}$. [2]

b) Obtain the inverse Laplace transform of $F(s)=\cot^{-1} s$. [3]

c) Find the value of $\Gamma\left(\frac{-1}{2}\right)$. [2]

d) Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\tan\theta}\ d\theta$ using Beta and Gamma functions. [3]

e) Evaluate $\iint_R \sqrt{x^2+y^2}\ dx\,dy$ by changing to polar coordinates, where R is the region in the $xy-$plane bounded by the circles $x^2+y^2=1$ and $x^2+y^2=4$. [2]

f) Find the value of the triple integral $\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^1 r^2 dr\, d\theta\, d\phi$. [3]

g) Find the normal vector and unit normal vector to the surface $z^2=x^2-y^2$ at $\left(2,1,\sqrt{3}\right)$. [2]

h) If $\vec{a}$ is a constant vector and $\vec{r}=x\hat{i}+y\,\hat{j}+z\,\hat{k}$, prove that $div\ (\vec{a}\times\vec{r})=0$. [3]

i) Evaluate $\int_c (x^2+yz)\,dz$, where c is given by $x=t, y=t^2,\ z=3t, 1\le t\le 2$. [2]

j) State Gauss's divergence theorem. [3]

PART-B

(50 Marks)

2.a) Find the Laplace transform of $f(t)=e^{-t}\left[\int_0^t \frac{\sin u}{u}\ du\right]$.

b) Find the Laplace transform of the periodic function $f(t)=t,\quad 0\le t\le a,\quad f(t+a)=f(t)$. [5+5]

OR

3.a) Obtain $L^{-1}\left\{\frac{s}{\left(s^2+1\right)^2}\right\}$ using convolution theorem.

b) Solve the initial value problem $y''+3y'+2y=e^{-t}$, $y(0)=0$, $y'(0)=-1$ using Laplace transforms. [5+5]

4.a) Evaluate $\int_0^{\infty} x^4 e^{-2x^2}\,dx$.

b) Prove that $\beta(m,n)=\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\,dx$. [5+5]

OR

5.a) Prove that $\beta(m,n)=2\int_0^{\pi/2} \sin^{2m-1}\theta\,\cos^{2n-1}\theta\,d\theta=\beta(m+1,n)+\beta(m,n+1)$.

b) Prove that $\int_0^1 x^m (\ln x)^n\,dx=\frac{(-1)^n\,n!}{(m+1)^{n+1}}$ where $m>1$ and n is a positive integer. [5+5]

6.a) Change the order of integration in $\int_0^1\int_{x^2}^{2-x} xy\,dx\,dy$ and hence evaluate the same.

b) Find the area of the region bounded by the parabolas $y^2=4x$ and $x^2=4y$. [5+5]

OR

7.a) Evaluate $\iiint_v (1-x)\,dx\,dy\,dz$, where v is the space in the first octant below the plane $3x+2y+z=6$.

b) Find the volume of the solid enclosed between the surfaces $x^2+y^2=4$ and $x^2+z^2=4$. [5+5]

8.a) Find the values of a and b so that the surface $ax^2-byz=a+2$ is orthogonal to the surface $4x^2y+z^3=4$ at (1, -1, 2).

b) Find the directional derivative of the scalar function $f(x,y,z)=xyz$ at (1, 4, 9) in the direction of the line from (1, 2, 3) to (1, -1, -3). [5+5]

OR

9.a) If $\vec{r}=x\,\hat{i}+y\,\hat{j}+z\,\hat{k}$ and $r=|\vec{r}|$, show that $\nabla\cdot\left(\frac{\vec{r}}{r^3}\right)=0$.

b) Prove that $curl\left(curl\ \vec{v}\right)=\nabla(\nabla\cdot\vec{v})-\nabla^2\vec{v}$. [5+5]

10.a) Show that $\int_c \left(2xy+3\right)dx+(x^2-4z)\,dy-4y\,dz$, where c is any path joining (0, 0, 0) to (1, -1, 3), does not depend on the path c and evaluate the integral.

b) Apply Stoke's theorem to evaluate $\oint_c (x+y)\,dx+(2x-z)\,dy+(y+z)\,dz$, where c is the boundary of the triangle with vertices (2, 0, 0), (0,3,0) and (0,0, 6). [5+5]

OR

11. Verify Green's theorem for $\oint_c e^{-x}\left(\cos y\,dx-\sin y\,dy\right)$, where c is the rectangle with vertices (0,0), $(\pi,0)$, $\left(\pi,\frac{\pi}{2}\right)$ and $\left(0,\frac{\pi}{2}\right)$. [10]

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