JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD B.Tech I Year II Semester Examinations, April - 2018

MATHEMATICS-II (Common to EEE, ECE, CSE, EIE, IT, ETM)

Time: 3 hours

Max. Marks: 75

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks) [2]

- 1.a) Find $L\{t^2 \ u(t-1)\}$.
 - b) Obtain the inverse Laplace transform of $F(s) = \cot^{-1} s$.

[3]

c) Find the value of $\Gamma\left(\frac{-1}{2}\right)$.

[2]

d) Evaluate $\int_{0}^{2} \sqrt{\tan \theta} \ d\theta$ using Beta and Gamma functions.

- [3]
- e) Evaluate $\iint_R \sqrt{x^2 + y^2} dx dy$ by changing to polar coordinates, where R is the region
 - in the xy-plane bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.
- [2]

f) Find the value of the triple integral $\int_{0}^{2\pi} \int_{0}^{\frac{\pi}{4}} \int_{0}^{1} r^{2} dr d\theta d\phi.$

- [3]
- g) Find the normal vector and unit normal vector to the surface $z^2 = x^2 y^2$ at $(2,1,\sqrt{3})$.
 - [2]
- h) If \vec{a} is a constant vector and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, prove that $div(\vec{a} \times \vec{r}) = 0$. [3]
- i) Evaluate $\int (x^2 + yz) dz$, where c is given by x = t, $y = t^2$, z = 3t, $1 \le t \le 2$.
- j) State Gauss's divergence theorem.

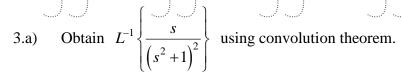
[3]

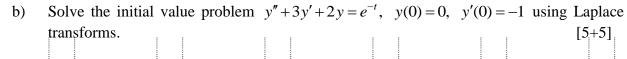
PART-B

(50 Marks)

- 2.a) Find the Laplace transform of $f(t) = e^{-t} \left[\int_{0}^{t} \frac{\sin u}{u} du \right]$.
 - b) Find the Laplace transform of the periodic function f(t) = t, $0 \le t \le a$, f(t+a) = f(t).

[5+5]





4.a) Evaluate
$$\int_{0}^{\infty} x^{4} e^{-2x^{2}} dx$$
.

b) Prove that
$$\beta(m,n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx$$
. [5+5]

5.a) Prove that
$$\beta(m,n) = 2 \int_{0}^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \ d\theta = \beta(m+1,n) + \beta(m,n+1)$$
.

b) Prove that
$$\int_{0}^{1} x^{m} (\ln x)^{n} dx = \frac{(-1)^{n} n!}{(m+1)^{n+1}} \quad \text{where } m > 1 \text{ and } n \text{ is a positive integer.}$$

6.a) Change the order of integration in
$$\int_{0}^{\infty} \int_{x^2}^{xy} dx dy$$
 and hence evaluate the same.

b) Find the area of the region bounded by the parabolas
$$y^2 = 4x$$
 and $x^2 = 4y$. [5+5] **OR**

7.a) Evaluate
$$\iiint_{v} (1-x) dx dy dz$$
, where v is the space in the first octant below the plane $3x+2y+z=6$.

b) Find the volume of the solid enclosed between the surfaces
$$x^2 + y^2 = 4$$
 and $x^2 + z^2 = 4$. [5+5]

8.a) Find the values of
$$a$$
 and b so that the surface $ax^2 - by z = a + 2$ is orthogonal to the surface $4x^2y + z^3 = 4$ at $(-1, -1, 2)$.

b) Find the directional derivative of the scalar function
$$f(x, y, z) = x y z$$
 at $(1, 4, 9)$ in the direction of the line from $(1, 2, 3)$ to $(1, -1, -3)$. [5+5]

9.a) If
$$\vec{r} = x \hat{i} + y \hat{j} + z \hat{k}$$
 and $r = |\vec{r}|$, show that $\nabla \cdot \left(\frac{\vec{r}}{\vec{r}^3}\right) = 0$.
b) Prove that $curl(curl \ \vec{v}) = \nabla(\nabla \cdot \vec{v}) - \nabla^2 \vec{v}$.

10.a)) Show that	$\int \int dx + \int (2xy + 3) $	$\int \int (x^2-4z) dy - 4y$	dz, where c is	s any path joining	(0, 0, 0) to
b)	Show that $\int_{c} (2xy+3)dx + (x^2-4z)dy - 4ydz$, where c is any path joining $(0,0,0)$ to $(1,-1,3)$, does not depend on the path c and evaluate the integral. Apply Stoke's theorem to evaluate $\oint_{c} (x+y)dx + (2x-z)dy + (y+z)dz$, where c is the boundary of the triangle with vertices $(2,0,0)$, $(0,3,0)$ and $(0,0,6)$.					
11.	Verify C	Green's theorem for $(0,0), (\pi,0), (\pi,\frac{\pi}{2})$	For $\oint_c e^{-x} \left(\cos y dx\right)$ $\left(0, \frac{\pi}{2}\right)$ and $\left(0, \frac{\pi}{2}\right)$.	$(x-\sin ydy)$, w	here c is the re	ectangle with
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