Code No: 133BC **R16**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech II Year I Semester Examinations, April/May - 2018
MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE
(Common to CSE, IT)

Time: 3 Hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

1.a) Construct the truth table for the following formula:
$\neg(P \vee (Q \wedge R)) \leftrightarrow ((P \vee Q) \wedge (P \vee R))$ [2]

b) Explain duality law. [3]

c) Give the formal definition for the composition of binary relations. [2]

d) What are the properties of a group? [3]

e) State addition principle and give an example of a problem solved by addition principle. [2]

f) State pigeon-hole principle. [3]

g) What is the general form of a first-order recurrence relation? [2]

h) What is the generating function of $1,-1,1,-1,\ldots$ [3]

i) If a simple graph G contains n vertices and m edges, how many number of edges are present in Graph G' (complement of G). [2]

j) How many edges are present in a complete graph with n vertices? Explain. [3]

PART- B

(50 Marks)

2.a) Show the following equivalence without constructing the truth table.
$((P \wedge Q \wedge A) \rightarrow C) \wedge (A \rightarrow (P \vee Q \vee C)) \Leftrightarrow (A \wedge (P \leftrightarrow Q)) \rightarrow C$

b) Without constructing a truth table, show that $A \wedge E$ is not a valid consequence of
$A \leftrightarrow B \quad B \leftrightarrow (C \wedge D) \quad C \leftrightarrow (A \vee E) \quad A \vee E$ [5+5]

OR

3.a) Obtain the principal disjunctive and conjunctive normal form of the following formula.
$(P \rightarrow (Q \wedge R)) \wedge (\neg P \rightarrow (\neg Q \wedge \neg R))$

b) For the following formulas, let the universe be $\mathbb{R}$. Translate each of the following sentences into a formula (using quantifiers):
i) There is a smallest number.
ii) Every positive number has a square root. (Do not use the square root symbol; use only multiplication.) [5+5]

4.a) Consider the following Hasse diagram of a partially ordered set $\langle P, R \rangle$, where $P = \{x_1, x_2, x_3, x_4, x_5\}$. Find the least and greatest members in P if they exist. Also find the maximal and minimal elements of P. Find the upper and lower bounds of $\{x_2, x_3, x_4\}$, $\{x_2, x_4, x_5\}$ and $\{x_1, x_2, x_3\}$. Also indicate the LUB and GLB of these subsets if they exist.

b) Let $n \in N^+$ and $G_1, G_2, ..., G_n$ be groups, and consider $\prod_{i=1}^{n} G_i := G_1 \times G_2 \times ... \times G_n = \{(a_1, a_2, ..., a_n) : a_i \in G_i \forall i = 1,2,...,n\}$ with the operation † where if $x = (a_1, a_2, ..., a_n)$ and $y = (b_1, b_2, ..., b_n)$, then $x \dagger y = (a_1 b_1, a_2 b_2, ..., a_n b_n)$, where each product $a_i b_i$ is performed according to the operation of the group G_i. Show that $\prod_{i=1}^{n} G_i$ is a group. [5+5]

OR

5.a) Find the transitive closure of the relation $R = \{(1,2), (2,3), (3,4), (4,1)\}$. Show R^i for all values of i that give new elements of the transitive closure.

b) Find all the subgroups of (i) $(Z_{12}, +_{12})$; and (ii) $(Z_7^*, \times_7)$. [5+5]

6. In the United States and Canada, a telephone number is a 10-digit number of the form $NXX - NXX - XXXX$ where $N \in \{2,3,..,9\}$ and $X \in \{0,1,2,...,9\}$. How many telephone numbers are possible? The first three digits of a telephone number are called an area code. How many different area codes must a city with 23,000,000 phones have? A previous scheme for forming a telephone numbers required a format of $NYX - NXX - XXXX$ where N and X are defined as above and Y is either a 0 or a 1. How many more phone numbers are possible under the new format than under the old format? [10]

OR

7.a) How many four letter words can be formed using the letters $a, a, a, b, b, c, c, c, c, d, d$?

b) Expand $(2x - y)^7$ using the Binomial Theorem. [5+5]

8.a) Solve the recurrence relation $a_n = 2a_{n-1} + 3a_{n-2}$ for $n \geq 2$ where $a_0 = 2$ and $a_1 = 2$.

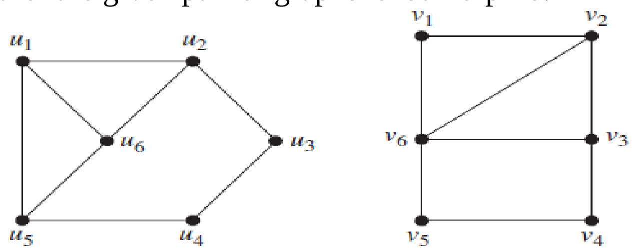
b) Using generating function find a_n in terms of n if $a_0 = 1, a_1 = \boldsymbol{2}$ and $a_{n+2} = 5a_{n+1} - 4a_n$ for $n \geq 0$. [5+5]

OR

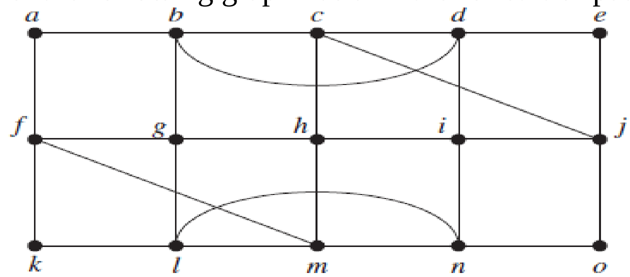
9.a) Solve the recurrence relation $T(n) = 4T(n-1) + 2^n$, with $T(0) = 6$.

b) Find the coefficient of x^{2005} in the generating function $\frac{1}{(1+5x)^2}$. [5+5]

10.a) Determine whether the given pair of graphs is isomorphic?



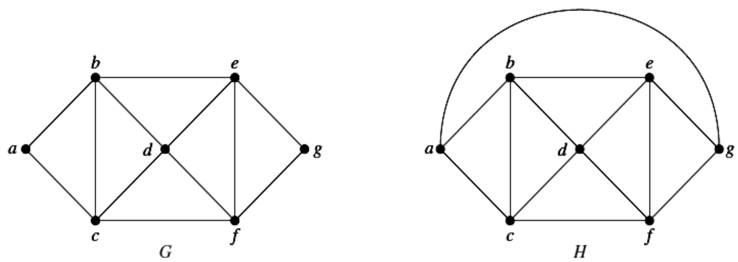
b) Determine whether the following graph has an Euler circuit or path. [5+5]



OR

11.a) How do you test the planarity of a graph? Explain.

b) What are the chromatic numbers of the graph *G* and *H*? [5+5]



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