# JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD 

## B.Tech II Year I Semester Examinations, May/June - 2019 MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE (Common to CSE, IT)

## Time: 3 Hours

Max. Marks: 75
Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have $\mathrm{a}, \mathrm{b}, \mathrm{c}$ as sub questions.

## PART- A

(25 Marks)
1.a) Find the negative of $p \rightarrow q$.
b) Test the validity of the following argument $p \wedge r \rightarrow \neg q, \neg q \rightarrow r \therefore p \wedge r \rightarrow r$
c) If $f(x)=x^{2}-6=y$, then find $f^{-1}(y)$.
d) If $f: G_{1} \rightarrow G_{2}$ is a homorphism and $a \in G$ then prove that $[f(a)]^{-1}=f\left(a^{-1}\right)$.
e) How many 5 digit numbers are possible, which are greater than 40000 with the digits $1,2,3,4,5$.
f) Find the number of positive integer solutions of $x+y+z=12$.
g) Solve the recurrence relation $u_{n+2}-u_{n+1}-6 u_{n}=0$.
h) Find the generating function of the sequence $1,3,3^{2}, 3^{3}$
i) If the adjacency matrix of the Graph is $\left[\begin{array}{llll}0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0\end{array}\right]$, then draw the graph.
j) If G is a k regular graph with 18 edges and the order of the graph is 9 . Find the value of k .

## PART - B

(50 Marks)
2.a) Test the validity of the following argument.

If I study, I will not fail in the examination.
If I do not watch TV in the evenings, I will study.
I failed in the examination.
Therefore I must watch TV in the evenings.
b) Prove that the following argument is valid.

$$
\begin{align*}
& \neg \exists x(p(x) \wedge q(x)) \\
& p(a) \\
& \therefore \quad \neg q(a) \tag{5+5}
\end{align*}
$$

## OR

3.a) Prove that $(p \uparrow q) \rightarrow r$ and $(p \wedge q) \vee r$ are logically equivalent.
b) Prove that the following argument is valid.
$\forall x p(x) \rightarrow \neg q(x)$
$\neg \exists x((r(x) \vee s(x)) \wedge \neg q(x))$
$r(a)$
$\therefore \quad \neg \mathrm{p}(\mathrm{a})$
4.a) Let $\mathrm{X}=\{1,2,3\}$ and $\mathrm{f}, \mathrm{g}$, h and s be functions from X to X given by $\mathrm{f}=\{(1,2),(2,3),(3,1)\}$, $\mathrm{g}=\{(1,2),(2,1),(3,3)\} \mathrm{h}=\{(1,1),(2,2),(3,1)\}$ Find fog, fohog.
b) If $f: G_{1} \rightarrow G_{2}$ is an isomorphism, then prove that $f^{-1}: G_{2} \rightarrow G_{1}$ is also an isomorphism.

## OR

5.a) Prove that the relation a congruent to b mood H is an equivalence relation.
b) Prove that the set of even integers forms a group under addition.
6.a) Find the number of solutions of $x_{1}+x_{2}+x_{3}=19$ with the condition $x_{1}>1, x_{2}>2, x_{3}>1$.
b) Prove that if 11 integers are selected from among $\{1,2, \ldots 20\}$, then the selection includes integer a and b such that $\mathrm{a}-\mathrm{b}=2$.

## OR

7.a) Find the number of integers < 250 and divisible by 3 or 5 or 11.
b) Suppose 14 students in a class appear at a university examination. Prove that there exists at least two among them whose seat number differ by a multiple of 13 .
8. Solve the recurrence relation. $u_{n}-2 u_{n-1}-3 u_{n-2}=5^{n}, n \geq 2, u_{0}=1, u_{1}=1$

## OR

9. Solve the recurrence relation using generating function. $u_{n+2}-2 u_{n+1}+u_{n}=2^{n}$, $u_{0}=2, u_{1}=1$.
10.a) Suppose that $G$ is a non directed graph with 12 edges. Suppose that $G$ has 6 vertices of degree 3 and the rest have degree less than 3. Determine the minimum number of vertices G can have.
b) Find the minimal spanning tree using Krushal's algorithm.


## OR

11.a) Show that the following graphs are isomorphic.

b) Prove that a graph $G$ with at least one edge is 2-chromatic if and only if $G$ has no cycle of odd length.

