

MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

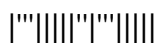
Common to Computer Science & Engineering (58) and Computer Science (05)

Time: 3 Hours

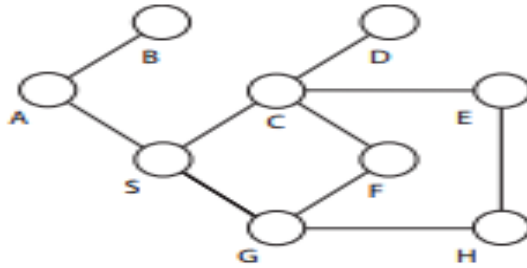
Max. Marks: 60

*Answer any FIVE Questions
All Questions Carry Equal Marks*

1. a i. Write the converse and contra-positive of the conditional statement: 6M
"If you obey the traffic rules, then you will not be fined".
ii. Prove without using truth table $\Lambda(P \rightarrow Q) \Rightarrow Q$.
- b Prove or disprove the validity of the following arguments using the rules of inference. i) All men are mortal ii) All kings are men iii) Therefore, all kings are mortal 6M
2. a Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $R = \{(x, y) / x+y \text{ is divisible by } 4\}$ in X . 6M
show that R is an Equivalence Relation.?
- b Let $f(x) = x^3 - 3x^2 + 2x + 3$. Find $f(x^2)$, $f(x+5)$ and $f(x^2-6)$? 6M
3. a Solve the following: 6M
i. Five red, two blue and 3 white balls are arranged in a row. If all the balls of the same colour are not distinguishable, how many different arrangements are possible?
ii. How many arrangements of all the letters in the word MISSISSIPPI have no consecutive S's?
- b State the Principle of Inclusion-Exclusion 6M
4. a Use iteration to solve the recurrence relation $a_n = a_{n-1} + n$ with $a_0 = 4$. 6M
b Suppose that r^n and q^n are both solutions to a recurrence relation of the form $a_n = \alpha a_{n-1} + \beta a_{n-2}$. Prove that $c \cdot r^n + d \cdot q^n$ is also a solution to the recurrence relation, for any constants c, d . 6M



5. a Construct the minimum cost spanning tree for the following graph using Depth first Search. 6M



- b Describe an algorithm to decide whether a graph is bipartite? 6M
6. a Solve the recurrence relation $a_n = 2a_{n-1} - a_{n-2}$. 6M
- i. What is the solution if the initial terms are $a_0 = 1$ and $a_1 = 2$?
 - ii. What do the initial terms need to be in order for $a_9 = 30$?
 - iii. For which x are there initial terms which make $a_9 = x$?
- b Use the Euclidean Algorithm to find $\text{GCD}(181, 587)$ 6M
7. a Prove that isomorphism is an equivalence relation on diagraphs? 6M
- b You have access to 1×1 tiles which come in 2 different colors and 1×2 tiles which come in 3 different colors. We want to figure out how many different $1 \times n$ path designs we can make out of these tiles. 6M
- a. Find a recursive definition for the sequence a_n of paths of length n .
 - b. Solve the recurrence relation using the Characteristic Root technique.
8. Discuss the following: 12M
- i. Hamiltonian graphs & Chromatic Numbers
 - ii. Binomial Coefficients
 - iii. Semi groups and Monoids
 - iv. homomorphism, Isomorphism

