

Total No. of Questions—8]

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[4957]-1005

**S.E. (Civil) (First Semester) EXAMINATION, 2016**

**ENGINEERING MATHEMATICS—III**

**(2012 PATTERN)**

**Time : Two Hours**

**Maximum Marks : 50**

**N.B. :—** (i) Answer Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4,  
Q. No. 5 or Q. No. 6, Q. No. 7 or Q. No. 8.

(ii) Figures to the right indicate full marks.

(iii) Non-programmable electronic pocket calculator is allowed.

(iv) Assume suitable data, if necessary.

(v) Neat diagrams must be drawn whenever necessary.

1. (a) Solve any *two* of the following : [8]

(i)  $(D^3 - 1)y = (1 + e^x)^2$

(ii)  $(D^2 - 6D + 9)y = \frac{e^{3x}}{x^2}$  (By method of variation of parameters)

(iii)  $[(2x + 1)^2 D^2 - 2(2x + 1)D - 12]y = 6x$ .

(b) Solve the following system of equations by Gauss elimination method : [4]

$$\left. \begin{aligned} 10x + 2y + z &= 9 \\ 2x + 20y - 2z &= -44 \\ -2x + 3y + 10z &= 22. \end{aligned} \right\}$$

P.T.O.

Or

2. (a) Solve the following system of equations in symmetrical form : [4]

$$\frac{dx}{x(2y^4 - z^4)} = \frac{dy}{y(z^4 - 2x^4)} = \frac{dz}{z(x^4 - y^4)}.$$

- (b) Solve : [4]

$$\frac{dy}{dx} = 1 + xy,$$

given that :

$$x_0 = 0, y_0 = 1, h = 0.1$$

and find  $y$  at  $x = 0.1$ , using modified Euler's method.

- (c) Solve the following system of equations by Cholesky's method : [4]

$$\left. \begin{aligned} 4x + 6y + 8z &= 0 \\ 6x + 34y + 52z &= -160 \\ 8x + 52y + 129z &= -452. \end{aligned} \right\}$$

3. (a) The first three moments of a distribution about the value 2 of a distribution are 1, 16 and  $-40$ . Find the mean, standard deviation and skewness of the distribution. [4]

- (b) An average box containing 10 articles is likely to have 2 defectives. If we consider a consignment of 100 boxes, how many of them are expected to have three or less defectives. [4]

- (c) Find the directional derivative of : [4]

$$\phi = xy^2 + yz^3,$$

at  $(2, -1, 1)$

along the line  $2(x - 2) = (y + 1) = z - 1$ .

Or

4. (a) Attempt any one : [4]

(i) Show that :

$$\nabla^2(r^2 \log r) = \frac{6}{r^2}.$$

(ii) Show that :

$$\bar{F} = \frac{\bar{a} \times \bar{r}}{r^n}$$

is solenoidal.

(b) Show that : [4]

$$\bar{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3z^2 - y)\hat{k}$$

is irrotational.

(c) If the two lines of regression are  $9x + y - \lambda = 0$  and  $4x + y = \mu$  and means of  $x$  and  $y$  are '2' and '-3' respectively, find the values of  $\lambda$  and  $\mu$  and the coefficient of correlation between  $x$  and  $y$ . [4]

5. (a) Find the work done in moving a particle from the point : [4]

$$\left(0, 1, \frac{\pi}{4}\right) \text{ to } \left(\pi, 2, \frac{\pi}{2}\right)$$

in the force field

$$\bar{F} = (y \sin z - \sin x)\hat{i} + (x \sin z + 2yz)\hat{j} + (xy \cos z + y^2)\hat{k}.$$

(b) Evaluate : [5]

$$\iint_S (x^2 y^3 \hat{i} + z^2 x^3 \hat{j} + x^2 y^3 \hat{k}) \cdot d\bar{S},$$

Where 'S' is the surface :

$$x^2 + y^2 + z^2 = a^2.$$

(c) Evaluate : [4]

$$\iint_S (\nabla \times \bar{F}) \cdot d\bar{S},$$

where  $\bar{F} = (2y + x)\hat{i} + (x - y)\hat{j} + (z - x)\hat{k}$ , and 'S' is the surface bounded by  $x = 0$ ,  $y = 0$ ,  $(x + y + z) = 1$ , which is not included in XOY-plane.

*Or*

6. (a) Evaluate  $\int_C \bar{F} \cdot d\bar{r}$ , using Green's theorem, where : [4]

$$\bar{F} = (2x^2 - y^2)\hat{i} + (x^2 + y^2)\hat{j},$$

and 'C' is the circle  $x^2 + y^2 = 1$ , above the x-axis.

(b) Using Gauss's divergence theorem, evaluate : [5]

$$\iint_S \bar{F} \cdot \hat{n} dS$$

where :

$$\bar{F} = x^3 \hat{i} + y^3 \hat{j} + z^3 \hat{k}$$

and 'S' is the surface of the sphere  $x^2 + y^2 + z^2 = 25$ .

(c) Using Stokes' theorem, evaluate : [4]

$$\int_C (y dx + z dy + x dz)$$

where 'C' is the curve of intersection of  $x^2 + y^2 + z^2 = a^2$ ,  $x + z = a$ .

7. (a) A string of length '2l' is fastened at both ends. The mid point of the string is taken to a height 'b' and then released from rest in that position. Obtain the displacement  $y(x, t)$  if : [7]

$$\frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 y}{\partial x^2}.$$

- (b) The equation for the conduction of heat along a bar of length 'l' is : [6]

$$\frac{\partial \theta}{\partial t} = k \frac{\partial^2 \theta}{\partial x^2}$$

Neglecting radiation, find an expression for  $\theta$  if the ends of the bar are maintained at zero temperature and if initially the temperature is 'T' at the centre of the bar and falls uniformly to zero at its ends.

*Or*

8. (a) A tightly stretched string with fixed ends  $x = 0$  and  $x = l$ , is initially at rest in its equilibrium position. If it is set vibrating, giving each point a velocity : [7]

$$3x(l - x) \text{ for } 0 < x < l,$$

find the displacement  $y(x, t)$ .

- (b) A rectangular plate with insulated surface is 10 cm wide and so long compared to its width, that it may be considered infinite in length without introducing an appreciable error. If the temperature along the short edge  $y = 0$  is given  $u(x, 0) = 100 \sin\left(\frac{\pi x}{10}\right)$ ,  $0 \leq x \leq 10$  while the two long edges  $x = 0$  and  $x = 10$  as well as other short edge are kept at  $0^\circ\text{C}$ , find the steady state temperature  $u(x, y)$ . [6]