Total No. of Questions—8] [Total No. of Printed Pages—6

Seat No.	

[5352]-105

S.E. (Civil) (I Sem.) EXAMINATION, 2018

ENGINEERING MATHEMATICS—III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Figures to the right indicate full marks.

(*iii*) Non-programmable electronic pocket calculator is allowed.

(*iv*) Assume suitable data, if necessary.

(*v*) Neat diagrams must be drawn wherever necessary.

1. (*a*) Solve any *two* of the following : [8]

(*i*) $\left(D^2 + 9\right)y = x^2 + 2x - \cos 3x$

(*ii*) $\left(x^2D^2 + 5xD + 3\right)y = \dfrac{\log x}{x^2}$

(*iii*) $\left(D^2 - 4D + 4\right)y = e^{2x}\sec^2 x$

(By method of variation of parameters.)

(*b*) Solve the following system of equation by Gauss-Seidel Iteration Method : [4]

$$10x_1 + x_2 + x_3 = 12$$
$$2x_1 + 10x_2 + x_3 = 13$$
$$2x_1 + 2x_2 + 10x_3 = 14.$$

P.T.O.

Or

2. (*a*) Weight 1 N, stretches a spring '5 cm'. A weight of '3 N' is attached to the spring and weight 'W' is pulled '10 cm' below the equilibrium position and then released. Determine the position and velocity as function of time. [4]

(*b*) Use Runge-Kutta method of fourth order to solve :

$$\frac{dy}{dx} = \frac{1}{x + y},$$

given that $x_0 = 0$, $y_0 = 1$, $h = 0.2$ therefore find y at $x = 0.4$. [4]

(*c*) Solve the following system of equation by Cholesky's Method : [4]

$$9x_1 + 6x_2 + 12x_3 = 17.4$$
$$6x_1 + 13x_2 + 11x_3 = 23.6$$
$$12x_1 + 11x_2 + 26x_3 = 30.8.$$

3. (*a*) The first four moments of a distribution about the value of '4' of the variable are –1.5, 17, –30 and 108. Find the moments about mean, skewness and kurtosis. [4]

(*b*) In certain examination, 200 students appeared in the subject of statistics. Average marks obtained were 50% with standard deviation 5%. How many students are expected to obtain more than 60% marks with the assumption that marks are distributed normally. Given that P(z = 2) = 0.4772. [4]

(*c*) Find the directional derivative of $\phi = xy^2 + yz^2$ at (2, −1, 1) in the direction of vector $2\hat{i} + \hat{j} + 3\hat{k}$. [4]

Or

4. (*a*) Attempt any *one* : [4]

(*i*) Show that :

$$\nabla^2 \left(\frac{\bar{a} \cdot \bar{b}}{r} \right) = 0,$$

where $\bar{a}, \bar{b}$ are constant vectors.

(*ii*) Show that :

$$\nabla \times \left[\bar{a} \times \left(\bar{b} \times \bar{r} \right) \right] = \bar{a} \times \bar{b}.$$

(*b*) Show that :

$$\bar{F} = \left(2xz^3 + 6y\right)\hat{i} + (6x - 2yz)\hat{j} + \left(3x^2z^2 - y^2\right)\hat{k}$$

is irrotational. Find the scalar potential ϕ such that $\bar{F} = \nabla\phi$. [4]

(*c*) Given :

x	x-series	y-series
Mean	18	100
Standard deviation	14	20

and coefficient of correlation is 0.8. Find the most probable value of y if $x = 70$ and most probable value of x if $y = 90$. [4]

5. (*a*) Find the work done by the force :

$$\bar{F} = \left(x^2 - yz\right)\hat{i} + \left(y^2 - xz\right)\hat{j} + \left(z^2 - xy\right)\hat{k},$$

in moving a particle from the point (1, 2, 1) to the point (2, –5, 7). [4]

(*b*) Use Gauss's Divergence theorem, to evaluate :

$$\iint_S \bar{F} \cdot \hat{n}\, ds$$

where

$$\bar{F} = y^2 z^2\ \hat{i} + x^2 z^2\ \hat{j} + x^2 y^2\ \hat{k},$$

and 'S' is the upper part of the sphere $x^2 + y^2 + z^2 = a^2$, above XOY-plane. [5]

(*c*) Evaluate :

$$\iint_S \left(\nabla \times \bar{F}\right) \cdot d\bar{s},$$

where

$$\bar{F} = (2x - y)\hat{i} - yz^2\ \hat{j} - y^2 z\ \hat{k},$$

and 'S' is the Hemi-sphere $x^2 + y^2 + z^2 = 1$, above XOY-plane. [4]

Or

6. (*a*) Evaluate :

$$\int_C \bar{F} \cdot d\bar{r},$$

where

$$\bar{F} = \cos y\, \hat{i} + x(1 - \sin y)\,\hat{j},$$

and 'C' is the Ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1,\ z = 0.$ [4]

(*b*) Evaluate :

$$\iint_S \left(x\hat{i} + y\hat{j} + z\hat{k}\right) \cdot d\bar{s},$$

using Gauss' Divergence theorem, where 'S' is the surface bounded by $x \geq 0$, $y \geq 0$, $z \geq 0$, $(x + y + z) \leq 1$. [5]

(*c*) Using Stokes' theorem, evaluate :

$$\int_C \left(x^2 + y - 4\right) dx + (3xy)\, dy + \left(2xz + z^2\right) dz,$$

where 'C' is the curve of intersection of $z = 9 - (x^2 + y^2)$ and $z \geq 0$. [4]

7. (*a*) An elastic string is stretched between two fixed points at a distance '*l*' apart. One end is taken at origin and, at a distance $\left(\frac{2l}{3}\right)$ from this end, the string is displaced at a distance '*a*' transversely and released from rest when in this position. Find $y(x, t)$ if y satisfies the equation :

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}.$$ [7]

(*b*) Solve :

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2},$$

for the conduction of heat along a rod without radiation, subject to conditions :

(*i*) $u(x \cdot t)$ is bounded at $t \to \infty$

(*ii*) $\frac{\partial u}{\partial x} = 0$ for $x = 0, \; x = l$

(*iii*) $u = lx - x^2$ for $t = 0, \; 0 \le x \le l$. [6]

Or

8. (*a*) If a string of length 'l' is initially at rest in its equilibrium position and each of its point is given a velocity $v(x)$ such that :

$$u(x) = ax, \qquad 0 \le x \le \frac{l}{2}$$
$$= a(l - x), \qquad \frac{l}{2} \le x \le l$$

Obtain the displacement $y(x, t)$ at any time 't'. [7]

(*b*) Solve $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$ with conditions :

(*i*) $v = 0$, as $y \to \infty \quad \forall \; x$

(*ii*) $v = 0$ at $x = 0, \quad \forall \; y$

(*iii*) $v = 0$ at $x = \pi, \quad \forall \; y$

(*iv*) $v = v_0$ at $y = 0, \quad 0 < x < \pi$. [6]