Total No. of Questions—**8**] [Total No. of Printed Pages—**4+1**

Seat No.	

[5252]-105

S.E. (Civil) (First Semester) EXAMINATION, 2017

ENGINEERING MATHEMATICS-III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Neat diagrams must be drawn wherever necessary.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.

(*iv*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* of the following : [8]

(*i*) $\frac{d^3y}{dx^3} - 7\frac{dy}{dx} - 6y = e^{2x}(1+x)$

(*ii*) $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2 \sin(\log x)$

(*iii*) Use the method of variation of parameters to solve the linear differential equation :

$$\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}.$$

(*b*) Solve the following system of linear equations by Gauss Elimination method : [4]

$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$
$$x + 2y + 9z = 34$$

P.T.O.

Or

2. (*a*) Solve the system of simultaneous symmetric equations : [4]

$$\frac{dx}{y-z} = \frac{dy}{z-x} = \frac{dz}{x-y}$$

(*b*) Apply Runge-Kutta method of 4th order to solve the differential equation : [4]

$$\frac{dy}{dx} = \frac{y-x}{y+x}, y(0) = 1$$

to find $y(1)$ with $h = 1$

(*c*) Solve the following system of equations by Cholesky method : [4]

$$2x - y = 1$$

$$-x + 3y + z = 0$$

$$y + 2z = 0$$

3. (*a*) Given the following information :

	Variable *x*	**Variable *y***
Arithmetic mean	8.2	12.4
Standard deviation	6.2	20

Coefficient of correlation between x and y is 0.9. Find the linear regression estimate of x, given $y = 10$. [4]

(*b*) A coin is so biased that apearance of head is twice likely as that of tail. If a throw is made 6 times, find the probability that at least 2 heads will appear. [4]

(*c*) If the directional derivative of $\phi = axy + byz + czx$ at (1, 1, 1) has maximum magnitude 4 in a direction parallel to x-axis, find the values of a, b, c. [4]

Or

4. (*a*) The first four moments of a distribution about the value 5 are 3, 30, 50 and 60. Obtain the first four central moments and coefficient of Skewness and Kurtosis. [4]

(*b*) Prove the following (any *one*) : [4]

(*i*) $\nabla\left(\dfrac{\bar{a}\,.\,\bar{r}}{r^n}\right)=\dfrac{\bar{a}}{r^n}-\dfrac{n(\bar{a}\,.\,\bar{r})}{r^{n+2}}\bar{r}$

(*ii*) $\nabla \times (\bar{r} \times \bar{u}) = \bar{r}\,(\nabla\,.\,\bar{u}) - (\bar{r}\,.\,\nabla)\bar{u} - 2\bar{u}$.

(*c*) Show that $\bar{\mathrm{F}} = (ye^{xy}\cos z)\bar{i} + (xe^{xy}\cos z)\bar{j} - e^{xy}\sin z\bar{k}$ is irrotational. Find corresponding scalar ϕ, such that $\bar{\mathrm{F}} = \nabla\phi$. [4]

5. (*a*) Find the work done in moving a particle along $x = a\cos\theta$, $y = a\sin\theta$, $z = b\theta$ from $\theta = \dfrac{\pi}{4}$ to $\theta = \dfrac{\pi}{2}$ under a field of force given by,

$$\bar{\mathrm{F}} = -3a\sin^2\theta\cos\theta\,\vec{i} + a(2\sin\theta - 3\sin^3\theta)\,\vec{j} + b\sin 2\theta\,\bar{k}\cdot$$ [4]

(*b*) Evaluate $\iint\limits_{\mathrm{S}}(\nabla\times\bar{\mathrm{F}}).d\bar{\mathrm{S}}$ for $\bar{\mathrm{F}} = y\bar{i} + z\bar{j} + x\bar{k}$ where S is the surface of paraboloid $z = 1 - x^2 - y^2$, $z \geq 0$ using Stokes' theorem. [4]

(*b*) Given that :

$$\iiint\limits_{\mathrm{V}}\frac{1}{r^2}\,d\mathrm{V} = \iint\frac{1}{r^2}\,\bar{r}.\,d\bar{\mathrm{S}},$$

where S is closed surface enclosing the volume V. Hence evaluate $\iint\limits_{\mathrm{S}}\dfrac{x\bar{i} + y\bar{j} + z\bar{k}}{r^2}, d\bar{\mathrm{S}}$ where S is surface of sphere $x^2 + y^2 + z^2 = a^2$. [5]

Or

6. (*a*) Using Green's theorem show that the area bounded by a simple closed curve C is given by $\dfrac{1}{2}\int(xdy - ydx)$ hence find the area of ellipse $x = a\cos\theta$, $y = b\sin\theta$. [4]

(*b*) Evaluate :

$$\iint_S (\nabla \times \overline{F}) . d\overline{S}$$

where $\overline{F} = (x^3 - y^3)\overline{i} - xyz\overline{j} + y^3\overline{k}$ and S is the surface $x^2 + 4y^2 + z^2 - 2x = 4$ above the plane $x = 0$. [4]

(*c*) Evaluate

$$\iint \overline{r} . \hat{n} dS,$$

over the surface of the sphere of radius 2 with centre at origin. [5]

7. (*a*) A homogeneous rod of conducting material of length 100 cm has its ends kept at zero temperature and the temperature initially is :

$$u(x, 0) = x, \quad 0 \leq x \leq 50$$
$$= 100 - x, \quad 50 \leq x \leq 100$$

Find the temperature $u(x, t)$ at any time. [6]

(*b*) A rectangular plate with insulated surface is 10 cm wide and so long to its width that it may considered infinite in length without introducing an appreciable error. If the temperature of short edge $y = 0$ is given by :

$$u(x, 0) = 20x \text{ for } 0 \leq x \leq 15$$
$$= 20 (10 - x), \text{ for } 5 \leq x \leq 10$$

and the two long edges $x = 0$, $x = 10$ as well as other short edge are kept at 0°C. Find the temperature $u(x, y)$ at any ponit (x, y). [7]

Or

8. (*a*) A tightly stretched string with fixed ends $x = 0$ and $x = l$ is initially in a position given by :

$y(x, 0) = y_0 \sin^3 \left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement y at any distance x from one end at any time t. [6]

(*b*) An infinitely long plane uniform plate is bounded by two parallel edges $x = 0$ and $x = \pi$ and an end rt. angles to them. The breadth of the plate is π. This end is maintained at temperature u_0 at all points and other edges at zero temperatures. Find steady state temperature function $u(x, y)$. [7]