Total No. of Questions—**8**] [Total No. of Printed Pages—**4+1**

Seat No.	

[4957]-1076

S.E. (Computer/IT) (II Semester) EXAMINATION, 2016

ENGG. MATHEMATICS–III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $(D^2 - 2D)y = e^x \sin x$

(*ii*) $\dfrac{dx}{3z-4y} = \dfrac{dy}{4x-2z} = \dfrac{dz}{2y-3x}$

(*iii*) $(D^2 + 9)y = x^2 + 2x + \cos 3x$.

(*b*) By considering Fourier cosine transform of $f(x) = e^{-mx}$ $(m > 0)$ prove that : [4]

$$\int_0^\infty \frac{\cos \lambda x}{\lambda^2 + m^2} d\lambda = \frac{\pi}{2m} e^{-mx}, \quad m > 0, x > 0.$$

P.T.O.

Or

2. (*a*) A capacitor 10^{-3} farad is in series with emf of 20 volts and an inductor of 0.4 H at $t = 0$, the charge Q and current I are zero, find Q at any time. [4]

(*b*) Find the inverse z transform (any *one*) : [4]

(*i*) $F(z) = \dfrac{1}{(z-2)(z-3)}$, by using inversion integral method.

(*ii*) $F(z) = \dfrac{1}{(z-3)(z-4)}, \quad |z| > 4$.

(*c*) Solve the following difference equation to find $f(k)$ [4]

$f(k + 2) + 3 f(k + 1) + 2 f(k) = 0$

$f(0) = 0, f(1) = 2, k \geq 0$.

3. (*a*) The first four moments about the working mean 3.5 of a distribution are 0.0375, 0.4546, 0.0609 and 0.5074. Calculate the first four moments about the mean. Also calculate the coefficient of skewness. [4]

(*b*) On an average a box containing 10 articles is likely to have 2 defectives. If we consider a consignment of 100 boxes, how many of them are expected to have three or less defectives ? [4]

(*c*) If the directional derivative of $\phi = axy + byz + czx$ at (1, 1, 1) has maximum magnitude 4 in a direction parallel to x-axis, find the values of *a*, *b*, *c*. [4]

Or

4. (*a*) Prove the following (any *one*) : [4]

(*i*) $\nabla\left(\frac{\overline{a}\,.\,\overline{r}}{r^2}\right) = \frac{\overline{a}}{r^2} - \frac{2(\overline{a}\,.\,\overline{r})}{r^4}\,\overline{r}$

(*ii*) $\nabla^2\left[\nabla\,.\left(\frac{\overline{r}}{r^2}\right)\right] = \frac{2}{r^4}$

(*b*) Show that the vector field $\overline{F} = f(r)\,\overline{r}$ is always irrotational. Also determine $f(r)$ such that the field $\overline{F}$ is solenoidal. [4]

(*c*) Calculate the coefficient of correlation from the following information :

$n = 10$, $\Sigma x = 40$, $\Sigma x^2 = 190$, $\Sigma y^2 = 200$, $\Sigma xy = 150$, $\Sigma y = 40$. [4]

5. (*a*) Evaluate $\oint_c \overline{F}\,.\,d\overline{r}$ where $\overline{F} = \sin z\hat{i} + \cos x\hat{j} + \sin y\hat{k}$, and C is the boundary of the rectangle $0 \le x \le \pi, 0 \le y \le 1$ and $z = 3$. [4]

(*b*) Use divergence theorem to evaluate :

$\iint_s (y^4 z^4 \hat{i} + z^4 x^4 \hat{j} + x^4 y^4 \hat{k}).\,d\overline{s}$, where *s* is the upper part of the sphere $x^2 + y^2 + z^2 = 4$ above the *xoy* plane. [4]

(*c*) Use Stokes' Theorem to evaluate $\int_C (5y\hat{i} + 2x\hat{j} + 7y\hat{k}) . d\bar{r}$, where C is the curve of intersection $x^2 + y^2 + z^2 = 4z$ and $x = z - 2$. [5]

Or

6. (*a*) Find the work done of $\overline{\text{F}}$ round the curve C, where $\overline{\text{F}} = y\hat{i} + z\hat{j} + x\hat{k}$ and C is the circle $x^2 + y^2 = 1$ and $z = 0$. [4]

(*b*) Using divergence theorem evaluate $\iint_s \overline{\text{F}} . \hat{n}\, ds$, where $\overline{\text{F}} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and s is all the surface, bounding the cube, $x = 0$, $x = 2$, $y = 0$, $y = 2$, $z = 0$ and $z = 2$. [5]

(*c*) Evaluate $\iint \text{curl}\, \overline{\text{F}} . \hat{n}\, ds$ for the surface of the paraboloid $z = 9 - (x^2 + y^2)$ above the *xoy* plane, where $\overline{\text{F}} = (x^2 + y - 4)\hat{i} + 3xy\hat{j} + (2xz + z^2)\hat{k}$. [4]

7. (*a*) If $V = r^n \sin n\theta$ find u such that $f(z) = u + iv$ is analytic and determine $f(z)$ in terms of z. [4]

(*b*) Evaluate $\oint_C \frac{2z^2 + 2z + 1}{(z+1)^3 (z-3)} dz$ where '*c*' is $|z - 3| = 2$. [5]

(*c*) Find the bilinear transformation which maps the points $-i$, 0, $2 + i$ of the z-plane on to the points 0, $2i$, 4 of w-plane. [4]

Or

8. (*a*) If $f(z)$ is an analytic function of z, and $f(z) = u + iv$ prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|\ \text{Re}\, f(z)|^2 = 2|\ \text{F}'(z)|^2$$ [4]

(*b*) Evaluate $\oint_{\text{C}} \frac{dz}{(z^2+4)^2}$ where 'C' is $|z - i| = 2$ [5]

(*c*) Show that the transformation $\text{W} = \frac{z-i}{z+i}$ maps the real axis in the z-plane into the circle $|\text{W}| = 1$. [4]