Total No. of Questions—**8**] [Total No. of Printed Pages—**6**

Seat No.	

[5352]-166

S.E. (Computer Engineering/Information Technology)

(II Sem.) EXAMINATION, 2018

ENGINEERING MATHEMATICS—III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

N.B. :— (*i*) Attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of electronic non-programmable calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve (any *two*) : [8]

(*i*) $\left(D^3 - 7D - 6\right) y = e^{2x}(1 + x)$

(*ii*) $\left(D^2 + 4\right) y = \dfrac{1}{1 + \cos 2x}$ by method of variation of parameters

(*iii*) $x^3 \dfrac{d^2 y}{dx^2} + 3x^2 \dfrac{dy}{dx} + xy = \sin(\log x)$.

(*b*) Find Fourier sine transform of : [4]

$$f(x) = \frac{e^{-ax}}{x}, \quad x > 0.$$

P.T.O.

Or

2. (*a*) A circuit consists of inductance L and condenser of capacity C in series. An emf $E \sin pt$ is applied at $t = 0$, the initial change and current being zero. If $p^2 = \frac{1}{LC}$, find current in the circuit at time t. [4]

(*b*) Find inverse z-transform of (any *one*) : [4]

(*i*) $F(z) = \frac{z^3}{(z-1)(z-2)^2}$, $|z| > 2$

(*ii*) $F(z) = \frac{10z}{(z-1)(z-2)}$ by inversion integral method.

(*c*) Solve difference equation : [4]

$f(k + 2) - 3f(k + 1) + 2f(k)$, $f(0) = 0$, $f(1) = 1$.

3. (*a*) The first four moments of a distribution about the value 4 are 1, 4, 10 and 45. Obtain the first four central moments, mean, standard deviation and coefficient of skewness and kurtosis. [4]

(*b*) A sample of 100 dry battery cells tested to find the length of life produced the following results; $\bar{X} = 12$ hours, $\sigma = 3$ hours. Assuming the data to be normally distributed, what percentage of battery cells are expected to have life between 10 and 14 hours ? [Given : For $z = 0.67$, area = 0.2487] [4] (*c*) If the directional derivative of : [4]

$$\phi = a(x + y) + b(y + z) + c(x + z)$$

has maximum value 12 in the direction of the line :

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-1}{3},$$

find the values of *a, b, c*.

Or

4. (*a*) Find the correlation coefficient for the following data : [4]

x	y
43	99
21	65
25	79
42	75
57	87
59	81

(*b*) Prove (any *one*) : [4]

(*i*) $\nabla^2\left[\nabla \cdot \left(\frac{\bar{r}}{r^2}\right)\right] = \frac{2}{r^4}$

(*ii*) $\nabla \times \left[\frac{1}{r}\left(r^2\bar{a} + (\bar{a} \cdot \bar{r})\,\bar{r}\right)\right] = 0.$

(*c*) Show that : [4]

$$\bar{F} = \left(2xz^3 + 6y\right) i + (6x - 2yz)\, j + \left(3x^2z^2 - y^2\right) k$$

is irrotational. Find the scalar potential associated with it.

5. (*a*) Evaluate : [4]

$$\int_C \overline{F} \cdot d\overline{r}$$

where

$$\overline{F} = \left(x^2 + xy\right)\overline{i} + \left(x^2 + y^2\right)\overline{j}$$

and C is the square formed by the lines $x = \pm 1$, $y = \pm 1$.

(*b*) Apply Stokes theorem to evaluate : [5]

$$\iint_S \left(\nabla \times \overline{F}\right) \cdot \hat{n} \, dS$$

where

$$\overline{F} = \left(x^3 - y^3\right)\overline{i} - xyz\,\overline{j} + y^3\,\overline{k}$$

and S is the surface $x^2 + y^2 + z^2 - 2x = 1$ above the plane $x = 0$.

(*c*) Show that : [4]

$$\iiint_V \frac{dV}{r^2} = \iint_S \frac{\overline{r} \cdot \hat{n}}{r^2} \, dS.$$

Or

6. (*a*) Using Green's theorem, evaluate : [4]

$$\int_C \overline{F} \cdot d\overline{r}$$

where

$$\overline{F} = x^2\,\overline{i} + xy\,\overline{j}$$

and C is the region enclosed by $y = x^2$ and the line $y = x$.

(*b*) Apply Stokes theorem to evaluate : [5]

$$\int_C \left(\left(x^2 + y^2\right)\bar{i} - 2xy\,\bar{j}\right)\cdot d\bar{r}$$

where C is the rectangle bounded by the lines $x = \pm a$, $y = 0$ and $y = b$.

(*c*) Evaluate : [4]

$$\iint_S 2x^2 y\, dy\, dz - y^2\, dz\, dx + 4xz^2\, dx\, dy$$

over the curved surface of the cylinder $y^2 + z^2 = 16$, bounded by $x = 0$ and $x = 2$.

7. (*a*) If $u = x^3 - 3xy^2$, find value of v such that $w = u + iv$ is an analytic function. Write w in term of z. [4]

(*b*) Evaluate : [4]

$$\oint_C \frac{2z^2 + z + 5}{\left(z - \frac{3}{2}\right)^2}\, dz$$

where C is the ellipse :

$$\frac{x^2}{9} + \frac{y^2}{16} = 1.$$

(*c*) Find the image of straight lines $x = 1$ and $y = 1$ under the transformation $w = z^2$. [5]

Or

8. (*a*) Show that real and imaginary parts of an analytic function are always harmonic. [4]

(*b*) Using Cauchy's residue theorem evaluate : [4]

$$\oint_C \frac{12z - 7}{(z-1)^2 (2z+3)} dz$$

where C is the circle :

$$|z + i| = \sqrt{3}.$$

(*c*) Find the bilinear transformation which maps the points $z = 1, i, 2i$ on the points $w = -2i, 0, 1$ respectively. [5]