Total No. of Questions—**8**] [Total No. of Printed Pages—**4+2**

Seat No.	

[4857]-1076

S.E. (Comp./I.T.) (Second Sem.) EXAMINATION, 2015

ENGINEERING MATHEMATICS-III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Attempt *four* questions : Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6** and Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of non-programmable electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $(D^2 - 2D - 3)y = 3e^{-3x} \sin e^{-3x} + \cos(e^{-3x})$

(*ii*) $(D^2 - 2D + 2)y = e^x \tan x$. (By variation of parameters)

(*iii*) $x^2 \dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} - 4y = x^2$.

(*b*) Find the Fourier transform of $e^{-|x|}$ and hence show that : [4]

$$\int_{-\infty}^{\infty} \frac{e^{i\lambda x}}{1+\lambda^2}\, d\lambda = \pi e^{-|x|}.$$

P.T.O.

Or

2. (*a*) An uncharged condenser of capacity C charged by applying an e.m.f. of value $E \sin \frac{t}{\sqrt{LC}}$ through the leads of inductance L and of negligible resistance. The charge Q on the place of condenser satisfied the differential equation : [4]

$$\frac{d^2Q}{dt^2} + \frac{Q}{LC} = \frac{E}{L} \sin \frac{t}{\sqrt{LC}} .$$

Prove that the charge at any time t is given by :

$$Q = \frac{EC}{2}\left[\sin \frac{t}{\sqrt{LC}} - \frac{t}{\sqrt{LC}} \cos \frac{t}{\sqrt{LC}}\right].$$

(*b*) Find the Inverse Z-transform (any *one*) : [4]

(*i*) $F(z) = \frac{z+2}{z^2 - 2z + 1}$ for $|z| > 1$.

(*ii*) $F(z) = \frac{10z}{(z-1)(z-2)}$ (Use inversion integral method).

(*c*) Solve the following difference equation to find $\{f(k)\}$: [4]

$$f(k+1) + \frac{1}{4} f(k) = \left(\frac{1}{4}\right)^k, \; k \geq 0, f(0) = 0 .$$

3. (*a*) The first four moments of a distribution about the value 4 are –1.5, 17, –30 and 108. Obtain the first four central moments, mean, standard deviation and coefficient of skewness and kurtosis. [4]

(*b*) If the probability that a concrete cube fails is 0.001. Determine the probability that out of 1000 cubes : [4]

(*i*) exactly two

(*ii*) more than one cubes will fail.

(*c*) Show that : [4]

$$\overline{F} = (y\sin z - \sin x)\overline{i} + (x\sin z + 2yz)\overline{j} + (xy\cos z + y^2)\overline{k}$$

is irrotational and hence find scalar function ϕ s.t. $\overline{F} = \nabla\phi$.

Or

4. (*a*) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at (2, –1, 2) along a line equally inclined with co-ordinate axes. [4]

(*b*) For a solenoidal vector field $\overline{F}$, show that : [4]

$$\text{curl curl curl curl } \overline{F} = \nabla^4\overline{F}.$$

(*c*) The regression equations are :

$$8x - 10y + 66 = 0 \quad \text{and} \quad 40x - 18y = 214.$$

The value of variance of x is 9. Find :

(*i*) The mean values of x and y

(*ii*) The correlation coefficient between x and y

(*iii*) The standard deviation of y. [4]

5. (*a*) Find the work done in moving a particle once round the ellipse : [4]

$$\frac{x^2}{16} + \frac{y^2}{4} = 1, \quad z = 0$$

under the field of force given by :

$$\overline{\mathrm{F}} = (2x - y + z)\,\overline{i} + (x + y - z^2)\,\overline{j} + (3x - 2y + 4z)\,\overline{k}\,.$$

(*b*) Evaluate : [4]

$$\iint_{\mathrm{S}} (\nabla \times \overline{\mathrm{F}}) \,.\, \hat{n}\; d\mathrm{S}$$

where $\overline{\mathrm{F}} = (x^3 - y^3)\,\overline{i} - xyz\,\overline{j} + y^2\,\overline{k}$

and S is the surface $x^2 + 4y^2 + z^2 - 2x = 4$ above the plane $x = 0$.

(*c*) Evaluate : [5]

$$\iint_{\mathrm{S}} \overline{\mathrm{F}} \,.\, \overline{d\mathrm{S}}$$

using divergence theorem, where

$$\overline{\mathrm{F}} = x^3\,\overline{i} + y^3\,\overline{j} + z^3\,\overline{k}$$

and S is the surface of sphere $x^2 + y^2 + z^2 = a^2$.

Or

6. (*a*) If [4]

$$\overline{\mathrm{F}} = x^2\,\overline{i} + (x - y)\,\overline{j} + (y + z)\,\overline{k}$$

displaces a particle from A(1, 0, 1) to B(2, 1, 2) along the straight line AB, find work done.

(*b*) Evaluate : [4]

$$\int_C (e^x dx + 2y dy - dz)$$

where C is the curve $x^2 + y^2 = 4,\ z = 2$.

(*c*) Evaluate : [5]

$$\iint_S \bar{F} \cdot \overline{dS}$$

using Gauss divergence theorem, where :

$$\bar{F} = 2xy\,\bar{i} + yz^2\,\bar{j} + xz\,\bar{k}$$

and S is the region bounded by :

$$x = 0,\ y = 0,\ z = 0,\ y = 3,\ x + 2z = 6.$$

7. (*a*) Show that $u = y^3 - 3x^2 y$ is harmonic function. Find its harmonic conjugate and the corresponding analytic function $f(z)$ in terms of z. [5]

(*b*) Using Cauchy's integral formula, evaluate : [4]

$$\int_C \frac{2z^2 + z + 5}{(z - 3/2)^2}\, dz$$

where C is $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

(*c*) Find the bilinear transformation which maps the points $z = 1, i, -1$, onto the points $w = 0, 1, \infty$. [4]

Or

8. (*a*) If $f(z)$ is an analytic function $v^2 = u$, then show that $f(z)$ is constant function. [4]

(*b*) Using residue theorem evaluate : [5]

$$\int_C \frac{z}{z^4 + 13z^2 + 36}\, dz$$

where 'C' is the circle $|z| = \frac{5}{2}$.

(*c*) Find the map of the circle $|z - i| = 1$ under the transformation $w = \frac{1}{z}$ into w-plane. [4]