Total No of Questions: [8] ***SEAT NO.***

[Total No. of Pages : 3]

S.E. 2012 (Electronics / E &TC)
Engineering Mathematics – III
(Semester - I)

Time: 2 Hours ***Max. Marks : 50***

Instructions to the candidates:

1) ***Answers Q1 or Q2, Q3 or Q4, Q5 or Q6, Q7 or Q8.***
2) ***Neat diagrams must be drawn wherever necessary.***
3) ***Figures to the right side indicate full marks.***
4) ***Use of Calculator is allowed.***
5) ***Assume Suitable data if necessary***

Q1) a) Solve (any two) [8]

i) $(D^2-1)y = x \sin x + (1+x^2)e^x$

ii) $d^2y/dx^2 + y = \operatorname{cosec} x$ (by variation of parameters)

iii) $x^2\ d^2y/dx^2 – 4x\ dy/dx + 6y = x^5$

b) Find Fourier cosine transform of the function [4]

$$f(x) = \begin{cases} \cos x & 0< x<a \\ 0 & x>a \end{cases}$$

OR

Q2) a) A resistance of 50 ohms, an inductor of 2 henries and farad capacitor are all in series with an e.m.f. of 40 volts . Find the instantaneous change and current after the switch is closed at t=0 , assuming that at that time the change on the capacitor is 4 coloumb. [4]

b) Solve (any one) [4]

i) Find z transform for $f(k) = (1/3)^{|k|}$

ii) Find inverse z transform of $[Z^2 / (Z-1/4) (Z-1/5)]$ for $|Z| <1/5$

c) Solve $f(k+2) + 3 f(k+1) +2 f(k) =0$ [4]

Given $f(0) =0$, $f(1)=1$

Q3) a) Solve the following differential equation to get y(0.1) [4]

$dy/dx = x-y^2$, $y(0) =1$

by using Runge- Kutta fourth order method. (h=0.1)

b) Find Lagrange’s long interpolating polynomial passing through set of points [4]

x	0	1	2
y	2	3	6

Use it to find y at x =1.5 and find $\int_0^2 y\,dx$.

c) Find the directional derivative of $\phi = 3\log(x+y+z)$ at (1,1,1) in the direction of tangent to the curve x= b sint, y=b cost, z=bt at t=0 [4]

OR

Q4) a) Show that (any one) [4]

i) $\nabla^2[\nabla\cdot(\bar{r}/r^2)] = 2/r^4$

ii) $\nabla(\bar{a}\cdot\bar{r}/r^3) = \bar{a}/r^3 - 3(\bar{a}\cdot\bar{r})/r^5\ \bar{r}$

b) If $\emptyset$,ψ satisfy Laplace equation then prove that the vector $(\emptyset\nabla\psi - \nabla\psi\,\emptyset)$is solenoidal. [4]

c) Use Simpsons 1/3rd rule to find [4]

$\int_0^{0.6} e^{-x^2}\,dx$ by taking seven ordinates

Q5) a) Find the work done by $\bar{F} = (2x + y^2)\,\hat{\boldsymbol{\imath}} + (3y-4x)\hat{\boldsymbol{\jmath}}$ in taking a particle around the parabolic arc $y = x^2$ from (0,0) to (1,1). [4]

b) Apply stoke's theorem to evaluate $\oint_c (4y\,dx + 2\,zdy + 6\,ydz)$ $where is\ curve\ of\ intersection\ of\ x^2+y^2+z^2 = 2z\ and\ z = x+1.$ [5]

c) Evaluate $\iint_s (2y\,\hat{\imath} + yz\hat{\jmath} + 2xz\hat{k}).d\bar{s}$ over the surface of region bounded by y=0, y=3, x=0, z=0,x+2z =6. [4]

OR

Q6) a) Using Green's Lemma, evaluate $\int_c \bar{F}\,.d\bar{r}$ where $\bar{F} = 3\,y\,\hat{\imath} + 2x\hat{\jmath}$ and c is boundary of region bounded by y=0,y=sinx for $0 \le x \le \Pi$ [4]

b) Evaluate [5]

$$\iint_s (z^2 - x)dydz - xydxdz + 3zdxdy$$

where s is closed surface of region bounded by $x=0, x=3, z=0, z=4-y^2$

c) Show that $\bar{E} = -\nabla\phi$ - 1/c $\frac{\partial \bar{A}}{\partial t}$; $\overline{H} = \nabla \times \bar{A}$ are solutions of Maxwell's equations [4]

i) $\nabla . \bar{H} = 0$ ii) $\nabla \times \bar{H}$ = 1/c $\frac{\partial \bar{E}}{\partial t}$ if $\nabla . \bar{A}$ + 1/c $\frac{\partial \phi}{\partial t}$

and $\nabla^2 \bar{A}$ = 1/c² $\frac{\partial^2 A}{\partial t^2}$

Q7) a) Show that the analytic function with constant amplitude is constant. [4]

b) By using Cauchy's integral formula, evaluate $\oint_c 2z^2 + z / z^2 - 1$ dz [5]

Where c is the circle |z-1| = 1

c) Find the bilinear transformation which maps the points z= -1,0,1 on the points [4]

W=0,i,3i of w-plane.

OR

Q8) a) If u= cos hx cosy then find the harmonic conjugate v such that f (z) = u+ iv is analytical function. [4]

b) Evaluate [5]

$\int_c \frac{12z-7}{(z-1)^2(2z+3)}$ dz where c is the circle |z| =2 using Cauchy's residue theorem.

c) Show that the transformation w=z+ 1/z – 2i maps the circle |z| =2 into an ellipse. [4]