Total No. of Questions—8] [Total No. of Printed Pages—7

Seat No.	

[4757]-1041

S.E. (Electronics/E & TC) (Second Semester)

EXAMINATION, 2015

ENGINEERING MATHEMATICS-III

(2012 Pattern)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6** and Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-2x}\sin 2x$

P.T.O.

(*ii*) $(D^2 - 4D + 4)y = e^{2x}x^{-2}$ (by variation of parameters)

(*iii*) $x^3 \dfrac{d^3y}{dx^3} + 2x^2 \dfrac{d^2y}{dx^2} + 2y = x + \dfrac{1}{x}$

(*b*) Solve : [4]

$$f(k) - 4f(k-2) = \left(\frac{1}{2}\right)^k, \; k \geq 0$$

Or

2. (*a*) The charge Q on the plate of condencer satisfies the differential equation : [4]

$$\frac{d^2Q}{dt^2} + \frac{Q}{LC} = \frac{E}{L} \sin \frac{t}{\sqrt{LC}}$$

Assuming $\dfrac{1}{LC} = \omega^2$ find the charge Q at any time '*t*'.

(*b*) Find the Fourier sine integral representation for the function : [4]

$$f(x) = \begin{cases} \dfrac{\pi}{2} \; ; \; 0 < x < \pi \\ 0 \; ; \; x > \pi \end{cases}$$

(*c*) Attempt any *one* : [4]

(*i*) Find *z*-transform of $f(k) = ke^{-3k}; \; k \geq 0$

(*ii*) Find :

$$z^{-1}\left[\frac{z^2}{z^2+1}\right]$$

3. (*a*) Given :

$$\frac{dy}{dx} = 3x + \frac{y}{2}; \ y(0) = 1 \quad h = 0.1$$

Evaluate $y(0.1)$ by using Runge-Kutta method of fourth order. [4]

(*b*) The distance travelled by a point *p* in XY – plane in a mechanism is given by *y* in the following table. Estimate distance travelled by *p* when $x = 4.5$.

x	***y***
1	14
2	30
3	62
4	116
5	198

[4]

(*c*) Find the directional derivative of function $\phi = xy^2 + yz^3$ at $(1, -1, 1)$ along the direction normal to the surface $2x^2 + y^2 + 2z^2 = 9$ at $(1, 2, 1)$. [4]

Or

4. (*a*) Prove that (any *one*) : [4]

(*i*) $\bar{a} \cdot \nabla\left[\bar{b} \cdot \nabla \frac{1}{r}\right] = -\frac{\left(\bar{a} \cdot \bar{b}\right)}{r^3} + \frac{3(\bar{b} \cdot \bar{r})(\bar{a} \cdot \bar{r})}{r^5}$

(*ii*) $\nabla \cdot \left[r \nabla \frac{1}{r^5}\right] = \frac{15}{r^6}$.

(*b*) Use Trapezoidal Rule to estimate the value of :

$$\int_0^2 \frac{x}{\sqrt{2 + x^2}} dx$$

by taking $h = 0.5$. [4]

(*c*) Show that the vector field $f(r)\bar{r}$ is always irrotational and then determine F(r) such that vector field $f(r)\bar{r}$ is solenoidal. [4]

5. (*a*) Evaluate :

$$\int_C \left[(2x^2y + y + z^2)i + 2(1 + yz^3)j + (2z + 3y^2z^2)k\right] . d\bar{r}$$

along the curve $C : y^2 + z^2 = a^2 \quad x = 0$ [4]

(*b*) Find

$$\iint_S \overline{F} \cdot \hat{n}\, ds.$$

where s is the sphere $x^2 + y^2 + z^2 = 9$ and

$\overline{F} = \left(4x + 3yz^2\right)\hat{i} - \left(x^2z^2 + y\right)\hat{j} + \left(y^3 + 2z\right)\hat{k}$ [4]

(*c*) Evaluate :

$$\iint_S \nabla \times \overline{F} \cdot \hat{n}\, ds$$

for the surface of the paraboloid $z = 4 - x^2 - y^2$; $(z \geq 0)$ and

$\overline{F} = y^2\hat{i} + z\hat{j} + xy\hat{k}$. [5]

Or

6. (*a*) Find the total work done in moving a particle is a force field $\overline{F} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$ along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ and $t = 2$. [5]

(*b*) Using divergence theorem to evaluate the surface integral $\iint_S \overline{F} \cdot \hat{n}\, ds$ where $\overline{F} = \sin xi + (2 - \cos x)j$ and S is the total surface area of the parallelopiped bounded by $x = 0$, $x = 3$, $y = 0$, $y = 2$, $z = 0$ and $z = 1$. [4]

(*c*) Equations of electromagnetic wave theory are given by :

(*i*) $\nabla . \bar{D} = \rho$

(*ii*) $\nabla . \bar{H} = 0$

(*iii*) $\nabla \times \bar{D} = \frac{-1}{C}\frac{\partial \bar{H}}{\partial t}$ and

(*iv*) $\nabla \times \bar{H} = \frac{1}{C}\left[\frac{\partial \bar{D}}{\partial t} + \rho \bar{v}\right]$

Prove that :

$$\nabla^2 \bar{D} - \frac{1}{C}\frac{\partial^2 \bar{D}}{\partial t^2} = \nabla\rho + \frac{1}{C^2}\frac{\partial}{\partial t}(\rho \bar{v})$$ [4]

7. (*a*) Find the analytic function $f(z) = u + iv$ if $2u + v = e^x(\cos y - \sin y)$. [5]

(*b*) Evaluate :

$$\int_C \frac{e^{2z}}{(z-1)(z-2)} dz,$$

where C is circle $|z| = 3$. [4]

(*c*) Find the bilinear transformation which maps the points $z = -1, 0, 1$ of z-plane into the points $w = 0, i, 3i$ of w-plane. [4]

Or

8. (*a*) Find the analytic function $f(z) = u + iv$

where

$u = r^3 \cos 3\theta + r \sin \theta.$ [4]

(*b*) Evaluate :

$$\int_C \frac{1 - 2z}{z(z - 1)(z - 2)} dz$$

where

C is $|z| = 1.5.$ [4]

(*c*) Find the map of the straight line $y = 2x$ under the transformation : [5]

$$w = \frac{z - 1}{z + 1}$$