Total No. of Questions—**8**] [Total No. of Printed Pages—**6**

Seat No.	

[5152]-140

S.E. (Electronics/E&TC) (Second Semester) EXAMINATION, 2017

ENGINEERING MATHEMATICS—III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagram must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of non-programmable pocket calculator (electronic) is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve (any *two*) : [8]

(*i*) $\left(D^2 + 2D + 1\right)y = xe^{-x}\cos x$

(*ii*) $\left(D^2 - 6D + 9\right)y = \dfrac{e^{3x}}{x^2}$

(*iii*) $x^2\dfrac{d^2y}{dx^2} - 4x\dfrac{dy}{dx} - 6y = x^5.$

P.T.O.

(*b*) Find Fourier transform of : [4]

$$f(x) = \begin{cases} x & |x| \le a \\ 0 & |x| > a \end{cases}$$

Or

2. (*a*) A resistance of 50 Ω, an inductance of 2 henries and a 0.005 farad capacitor is in series with an e.m.f. of 40 volts and an open switch. Find the instantaneous charge and current after the switch is closed at time $t = 0$, assuming that at that time the charge on the capacitor is 4 coulomb. [4]

(*b*) Solve (any *one*) : [4]

(*i*) Find z-transform of $f(k) = k5^k,\ k \ge 0$.

(*ii*) Find inverse z-transform of : [4]

$$\frac{z^2}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{3}\right)}, |z| > \frac{1}{2}.$$

(*c*) Solve : [4]

$f(k + 2) + 3f(k + 1) + 2f(k) = 0,\ f(0) = 0,\ f(1) = 1.$

3. (*a*) Solve the different equation $\frac{dy}{dx} = \frac{1}{x + y}$ using Runge-Kutta fourth order method given that $y(0) = 1$ to find y at $x = 0.2$ taking $h = 0.2$. [4]

(*b*) Find Lagrange's interpolating polynomial satisfying the data : [4]

x	y
0	3
1	5
3	15
4	35

(*c*) In what direction from the point (2, 1, –1) is the directional derivative of $\phi = x^2yz^3$ a maximum ? What is the magnitude of this maximum ? [4]

Or

4. (*a*) Show that (any *one*) : [4]

(*i*) $\nabla^2\left(r^2 \log r\right) = 5 + 6 \log r$

(*ii*) $\nabla \times \left[\bar{a} \times \left(\bar{b} \times \bar{r}\right)\right] = \bar{a} \times \bar{b}$.

(*b*) Show that : [4]

$$\bar{F} = \left(6xy + z^3\right)\bar{i} + \left(3x^2 - z\right)\bar{j} + \left(3xz^2 - y\right)\bar{k}$$

is irrotational. Find scalar potential ϕ such that $\bar{F} = \nabla\phi$.

(*c*) Evaluate : [4]

$$\int_1^2 \frac{dx}{x^2}$$

using Simpson's $\left(\frac{1}{3}\right)$rd rule, taking $h = 0.25$.

5. (*a*) Evaluate : [5]

$$\int_C \bar{F} \cdot d\bar{r}$$

where $\bar{F} = z\bar{i} + x\bar{j} + y\bar{k}$ and C is the arc of the curve $x = \cos t$, $y = \sin t$, $z = t$ from $t = 0$ to $t = \pi$.

(*b*) Evaluate : [4]

$$\iint_S \left(\nabla \times \bar{F}\right) \cdot \hat{n}\ dS$$

where $\bar{F} = \left(x^3 - y^3\right)\bar{i} - xyz\,\bar{j} + y^2\bar{k}$ and S is the surface $x^2 + 4y^2 + z^2 - 2x = 4$ above the plane $x = 0$.

(*c*) If $\bar{E} = \nabla\phi$ and $\nabla^2\phi = -4\pi\rho$ prove that : [4]

$$\iint_S \bar{E} \cdot d\bar{S} = -4\pi \iiint_V \rho dV.$$

Or

6. (*a*) Using Green's Theorem evaluate : [5]

$$\int_C \left(\frac{1}{y}dx + \frac{1}{x}dy\right)$$

where C is the boundary of the region bounded by the parabola $y = \sqrt{x}$ and lines $x = 1$ and $x = 4$.

(*b*) Using Stokes' Theorem, evaluate : [4]

$$\int_C \bar{F} \cdot d\bar{r}$$

where $\overline{F} = 3y\overline{i} + 2x\overline{j}$ and C is the boundary of the rectangle $0 \le x \le \pi$, $0 \le y \le 1$ and $z = 3$.

(*c*) Prove that : [4]

$$\oint_C (\overline{a} \times \overline{r}) \cdot d\overline{r} = 2\overline{a} \cdot \iint_S d\overline{S}$$

where S is any open surface with boundary C.

7. (*a*) If $f(z) = u + iv$ is an analytic function with $v = 3x^2y - y^3$, find u and express $f(z)$ in terms of z. [4]

(*b*) Evaluate : [4]

$$\oint_C \frac{4z^2 + z}{z^2 - 1} dz$$

where C is $|z - 1| = \frac{1}{2}$.

(*c*) Find the bilinear transformation which maps the points $-1, 1, 0$ from z-plane into the points $0, 3i, i$ of the W-plane. [5]

Or

8. (*a*) Prove that an analytic function with constant argument is constant. [4]

(*b*) Evaluate : [4]

$$\oint_C \frac{z^3 - 5}{(z+1)^2 (z-2)} dz$$

where C is $|z| = \frac{3}{2}$.

(*c*) Show that the transformation $W = z + \frac{1}{z} - 2i$ maps the circle $|z| = 2$ onto an ellipse. [5]