Total No. of Questions—**8**] [Total No. of Printed Pages—**7**

Seat No.	

[4657]-541

S.E. (Electronics/E&TC) (Second Sem.) EXAMINATION, 2014

ENGINEERING MATHEMATICS–III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Answer Q. **1** or Q. **2**, Q. **3** or Q. **4**, Q. **5** or Q. **6** and Q. **7** or Q. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of non-programmable electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $(D^2 - 2D)\, y = e^x \sin x$ by method of variation of parameters.

(*ii*) $x^2 \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + 4y = \cos(\log x) + x\sin(\log x)$

(*iii*) $(D^2 - 2D + D)y = x\, e^x \sin x$.

P.T.O.

(*b*) Find Fourier sine transform of : [4]

$$f(x) = x^2 \quad , \quad 0 \le x \le 1$$
$$= 0 \quad , \quad x > 1 .$$

Or

2. (*a*) An electric current consists of an inductance 0.1 henry, a resistance R of 20 ohms and a condenser of capacitance C of 25 microfarads. If the differential equation of electric circuit is :

$$L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = 0$$

then find the at time t, given that at $t = 0$, $q = 0.05$ coulombs $\frac{dq}{dt} = 0$. [4]

(*b*) Solve (any *one*) : [4]

(*i*) Find z transform of :

$$f(k) = \frac{2^k}{k}, \quad k \ge 1 .$$

(*ii*) Find inverse z transform :

$$F(z) = \frac{1}{(z-3)(z-2)}, \quad |z| < 2 .$$

(*c*) Solve : [4]

$$12f(k+2) - 7f(k+1) + f(k) = 0, \quad k \geq 0,$$

$$F(0) = 0, \quad F(1) = 3.$$

3. (*a*) Solve the following differential equation to get $y(0.2)$: [4]

$$\frac{dy}{dx} = \frac{1}{x+y}, \quad y(0) = 1, \; h = 0.2$$

by using Runge-Kutta fourth order method.

(*b*) Find Lagrange's interpolating polynomial passing through set of points : [4]

x	***y***
0	4
1	3
2	6

Use it to find y at $x = 2$, $\frac{dy}{dx}$ at $x = 0.5$ and $\int_0^3 y \, dx$.

(*c*) Find the directional derivative of : [4]

$$\phi = 5x^2y - 5y^2z + 2z^2x$$

at the point (1, 1, 1) in the direction of the line :

$$\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}.$$

Or

4. (*a*) Show that (any *one*) : [4]

(*i*) $\nabla\left(\frac{\bar{a}\,.\,\bar{r}}{r^n}\right) = \frac{\bar{a}}{r^n} - \frac{n(\bar{a}\,.\,\bar{r})}{r^{n+2}}\,\bar{r}$

(*ii*) $\nabla^2 f(r) = \frac{d^2 f}{dr^2} + \frac{2}{r}\,\frac{df}{dr}$.

(*b*) Find the function $f(r)$ so that $f(r)\,\bar{r}$ is solenoidal. [4]

(*c*) Evaluate : [4]

$$\int_0^1 \frac{dx}{1+x^2}$$

using Simpson's $\frac{3}{8}$ rule taking $h = \frac{1}{6}$.

5. (*a*) Find the work done by the force : [4]

$$(2xy + 3z^2)\,\bar{i} + (x^2 + 4yz)\,\bar{j} + (2y^2 + 6xz)\,\bar{k}$$

in taking a particle from (0, 0, 0) to (1, 1, 1).

(*b*) Apply Stokes' theorem to calculate : [5]

$$\int_c (4y\,dx + 2z\,dy + 6y\,dz)$$

where c is the curve of intersection of $x^2 + y^2 + z^2 = 6z$, $z = x + 3$.

(*c*) Evaluate : [4]

$$\iint_s (xz^2\, dydz + (x^2y - z^2)\, dzdx + (2xy + y^2z)\, dxdy)$$

where s is the surface enclosing a region bounded by hemisphere $x^2 + y^2 + z^2 = 4$ above xoy plane.

Or

6. (*a*) If [4]

$$\overline{F} = \frac{1}{x^2 + y^2}(-y\,\overline{i} + x\,\overline{j})$$

then show that :

$$\oint_c \overline{F} \cdot d\overline{r} = 2\pi,$$

where c is circle $x^2 + y^2 = 1$.

(*b*) Evaluate : [5]

$$\iint_s (4xz\,\overline{i} - y^2\,\overline{j} + yz\,\overline{k}) \cdot d\overline{s}$$

over the cube bounded by the planes :

$$x = 0,\ x = 2,\ y = 0,\ y = 2,\ z = 0,\ z = 2.$$

(*c*) Maxwell's electromagnetic equations are : [4]

$$\nabla \cdot \overline{B} = 0, \quad \nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}.$$

Given $\bar{B} = \text{curl } \bar{A}$ then deduce that :

$$\bar{E} + \frac{\partial \bar{A}}{\partial t} = -\text{grad V}$$

where V is the scalar point function.

7. (*a*) Show that : [5]

$$u = e^{-x}(x \sin y - y \cos y)$$

is harmonic and determine an analytic function $f(z) = u + iv$.

(*b*) Evaluate : [4]

$$\int_c (z - z^2)\, dz$$

where c is the upper half of the unit circle $|z| = 1$.

(*c*) Find the Bilinear transformation which maps the points $z = 0, -1, \infty$ in the z-plane onto the points $w = -1, -(2 + i), i$ in the w-plane. [4]

Or

8. (*a*) Find the analytic function $f(z) = u + iv$ if : [4]

$$v = (r - 1/r)\sin\theta,\ r \neq 0.$$

(*b*) Using Cauchy's integral formula, evaluate the integral : [5]

$$\int_c \frac{(z+4)}{(z^2+2z+5)}\,dz$$

where c is the curve $|z+1-i|=2$.

(*c*) Find the image in the w-plane of the circle $|z-3|=2$ in the z-plane under the inverse mapping $w=\frac{1}{z}$. [4]