Total No. of Questions—**8**] [Total No. of Printed Pages—**5**

Seat No.	

[5057]-241

S.E. (Electronics/E&TC) (Second Semester) EXAMINATION, 2016

ENGINEERING MATHEMATICS-III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of non-programmable pocket calculator (electronic) is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve (any *two*) : [8]

(*i*) $(D^2 - 2D + 1)y = x\, e^x \sin x$

(*ii*) $\dfrac{d^2y}{dx^2} + y = \tan x$

(*iii*) $(1+x)^2 \dfrac{d^2y}{dx^2} + (1+x)\dfrac{dy}{dx} + y = 4 \cos [\log (1 + x)]$

(*b*) Find Fourier sine transform of [4]

$$f(x) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & x > 1 \end{cases}$$

P.T.O.

Or

2. (*a*) An uncharged condenser of capacity C charged by applying e.m.f. of value $E\sin\frac{1}{\sqrt{LC}}$ through the leads of inductance L and negligible resistance. The charge Q on the plate of condenser satisfies the differential equation $\frac{d^2Q}{dt^2}+\frac{Q}{LC}=\frac{E}{L}\sin\frac{t}{\sqrt{LC}}$, Prove that the charge at any time t is given by :

$$Q=\frac{EC}{2}\left[\sin\frac{t}{\sqrt{LC}}-\frac{t}{\sqrt{LC}}\cos\frac{t}{\sqrt{LC}}\right]$$ [4]

(*b*) Solve (any *one*) : [4]

(*i*) Find z-transform of $f(x) = 4^k \sin(2k + 3)$.

(*ii*) Find inverse z-transform of $\frac{1}{(z-1)(z-2)}, |z| > 2$

(*c*) Solve : [4]

$12f(k + 2) - 7f(k + 1) + f(k) = 0, k \geq 0, f(0) = 0.$

3. (*a*) Find a polynomial passing through the points (0, 1), (1, 1), (2, 7), (3, 25), (4, 61), (5, 121) using Newton's interpolation formula and hence find the value of the polynomial at $x = 0.5$. [4]

(*b*) Solve the equation $\frac{dy}{dx}=1+xy; y(0)=1$ to find y at $x = 0.1$ using modified Euler's method taking $h = 0.1$ correct upto four decimal places. [4]

(*c*) Find the directional derivative of $\phi = xy^2 + yz^3$ at $(1, -1, 1)$ towards the point $(2, 1, -1)$. [4]

Or

4. (*a*) Show that (any *one*) : [4]

(*i*) $\nabla \cdot \left[r \nabla \left(\frac{1}{r^n} \right) \right] = \frac{n(n-2)}{r^{n+1}}$

(*ii*) $\nabla \left(\frac{\overline{a} \cdot \overline{r}}{r^4} \right) = \frac{\overline{a}}{r^4} - \frac{4(\overline{a} \cdot \overline{r})}{r^6} \overline{r}$.

(*b*) Show that vector field $\overline{F} = (x^2 - yz)\overline{i} + (y^2 - zx)\overline{j} + (z^2 - xy)\overline{k}$ is irrotational. Find scalar potential ϕ such that $\overline{F} = \nabla\phi$. [4]

(*c*) Evaluate $\int_0^3 \frac{dx}{1+x}$ by using Simpson's $\frac{3}{8}$ th rule by taking 7 ordinates. [4]

5. (*a*) Find the work done in moving a particle once round the ellipse

$$\frac{x^2}{16} + \frac{y^2}{4} = 1, \quad z = 0$$

under the field of force given by

$$\overline{F} = (2x - y + z)\overline{i} + (x + y - z)\overline{j} + (3x - 2y + 4z)\overline{k}.$$ [4]

(*b*) Evaluate $\iint_S (\nabla \times \overline{F}) \cdot d\overline{S}$ for $\overline{F} = y\overline{i} + z\overline{j} + x\overline{k}$ where S is the surface of paraboloid $z = 9 - x^2 - y^2$, $z \geq 0$. [4]

(c) Evaluate :

$$\iint_S \bar{F} \cdot d\bar{S}$$

using divergence theorem, where, $\bar{F} = x^3\bar{i} + y^3\bar{j} + z^3\bar{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. [5]

Or

6. (a) Find work done in moving a particle in the force field $\bar{F} = 3x^2\bar{i} + (2xz - y)\bar{j} + z\bar{k}$ along the curve

$$x^2 = 4y; \qquad 3x^3 = 8z$$

from $x = 0$ to $x = 2$. [4]

(b) Evaluate :

$$\oint_C (e^x dx + 2y\, dy - dz)$$

where C is the curve $x^2 + y^2 = 4$, $z = 2$. [4]

(c) Evaluate :

$$\iint_S \bar{F} \cdot d\bar{S}$$

using Gauss divergence theorem where $\bar{F} = 2xy\,\bar{i} + yz^2\bar{j} + xz\bar{k}$ and S is the region bounded by

$$x = 0,\ y = 0,\ z = 0,$$

$$y = 3,\ x + 2z = 6.$$ [5]

7. (a) If $f(z) = u + iv$ is analytic and $v = \frac{-y}{x^2 + y^2}$, find $f(z)$ in terms of z. [4]

(*b*) Evaluate $\oint_C \frac{\sin^2 z}{\left(z - \frac{\pi}{6}\right)^3} dz$, where C is $|z| = 1$. [4]

(*c*) Find the bilinear tranformation which maps the points –2, 0, 2 from *z*-plane into the points 0, *i*, –*i* of the *w*-plane. [5]

Or

8. (*a*) If $f(z)$ is an analytic function, show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2$. [4]

(*b*) Evaluate $\oint_C \frac{z+2}{z^2+1} dz$, where C is $|z - i| = \frac{1}{2}$. [4]

(*c*) Show that under the transformation $w = \frac{i-z}{i+z}$, *x*-axis in the *z*-plane is mapped onto the circle $|w| = 1$. [5]