Total No of Questions: [08] **SEAT** [ ]

[Total No. of Pages : *3*]

S.E. (Electrical/ Instrumentation) (Engg Mathematics - III)
(2012 Pattern) (Semester - I)

Time: 2 Hours ***Max. Marks : 50***

Instructions to the candidates:

1) ***Attempt Q 1 or Q2, Q3 or Q4, Q5 or Q6, Q7 or Q8.***
2) ***Neat diagrams must be drawn wherever necessary.***
3) ***Figures to the right side indicate full marks.***
4) ***Use of Calculator is allowed.***
5) ***Assume Suitable data if necessary***

SECTION I

Q1) a) Solve Any TWO [08]

i) $\frac{d^2y}{dx^2} - 4y = (1+ e^x)^2 + xe^x$

ii) $\frac{d^3y}{dx^3} - 4 = sin2x$

iii) y'' – 6y' + 9y = $\frac{e^{3x}}{x^2}$ using method of variation of Parameters

b) Solve using Laplace Transform, [04]
y'' + y = t , given y(0) = 1, y'(0) = -2

OR

Q2) a) Find an Electric circuit, the charge Q on the plate of the condenser is given by [04]
$L\frac{d^2Q}{dt^2}$ + R $\frac{dQ}{dt} + \frac{Q}{C} = E\ sin\omega t$ where $\omega^2 = \frac{1}{LC}$, find the charge Q at any time t when Q=0 at time t=0

b) Solve any one: [04]

i) Find Laplace Transform of ($\frac{cosat-cosbt}{t}$)

ii) Find Inverse Laplace transform of
F(s) = $\frac{4s+15}{16\,s^2-25}$

c) Solve [04]

$$\mathcal{L}\, t.U\,(t-4) - t^3\delta(t-2)$$

Q3) a) Find the Fourier cosine transform of [04]

$$f(x)=\begin{cases} x & ;0<x<1/2 \\ 1-x & ;1/2<x<1; \\ 0 & ;x>1 \end{cases}$$

b) Attempt any one: [04]

i) Find the Z transform of f(k) = (k+3) 2^k

ii) Find the inverse Z transform of

$$f(z)=\frac{z(z+1)}{z^2-2z+1}$$

c) Find the directional derivative of $\varphi = axy + byz + czx, at$ (1,1,1) has maximum magnitude '4' in the direction parallel to X=axis, find the value of a,b,c [04]

OR

Q4) a) Show that any one of the following [04]

i) $\nabla.[\mathrm{r}\nabla(\frac{1}{r^4})]=\frac{8}{r^5}$

ii) $\bar{a}.\nabla[\bar{\mathrm{b}}.\nabla(\frac{1}{r})]=\frac{3(\bar{a}.\bar{r})(\bar{\mathrm{b}}.\bar{r})}{r^5}-\frac{(\bar{a}.\bar{\mathrm{b}})}{r^3}$

b) Show that, $\bar{F}=f(r)\bar{r}$ is irrational. Find $f(r)$ such that $\bar{F}$ is solenoidal. [04]

c) Solve $f(k+1)+(\frac{1}{2})^k, k\geq 0, f(0)=0$ [04]

Q5) a) If $\bar{F}=(2Y+3)\mathrm{i}+\mathrm{xzj}+(\mathrm{yz}-\mathrm{x})\mathrm{k}$ evaluate $\int_c \bar{F}.d\bar{r}$ where C is the straight line joining (0,0,0) and (2,1,1) [04]

b) Evaluate $\iint_s \bar{F}.\hat{n}\ ds$ for $\bar{F}=xyi+yzj+z^2k$ over the surface of the cube with side a and open in xy plane. [04]

Q6) a) Evaluate $\int_c \bar{F}.d\bar{r}$ for $\bar{F}=3x^3i+(2\,\mathrm{xz}-\mathrm{y})\,\mathrm{j}\,y+2k$ and C is the ellipse $x^2+2y^2=1,\ \mathrm{z=0}$ in the quadrant. [04]

b) Using Stoke's theorem, evaluate $\int_c \bar{F}.d\bar{r}$ where $\bar{F}=y^2i+x^2j-(x+z)k$ and C is the boundary of triangle with vertices (0,0,0), and (1,0,0) and (1,1,0) [05]

c) Evaluate $\iint_s \bar{F}.\hat{n}\, ds$ for $\bar{F} = y^2 z^2 i + x^2 z^2 k$ where S is the hemisphere $x^2 + y^2 + z^2 = 9$ above xy plane. [04]

Q7) a) If $u = e^x \cos y + x^2 - y^2$, find its harmonic conjugate v, write f(z) = u+iv in terms of z. [04]

b) Evaluate the integral [05]

$\int_{1-i}^{2+i} (2z+4)\,dz$, along the path x=t+1 and y=2t^2-1

c) Find the bilinear transformation which maps points 1, i, 2i of z plane onto the points -2i, 0, 1 of the ω- plane [04]

OR

Q8) a) Show that, f(z) = $\sqrt{xy}$ is not analytic function even though C R equations satisfied at origin [04]

b) Evaluate [05]

$$I = \oint_c \frac{z^2 - 5}{(z+1)^2 (z-2)} dz$$ where C is contour |z| = 3

c) Show that, [04]

$$w = \frac{2z+3}{z-4}$$ transforms circle $x^2 + y^2 - 4x = 0$ into straight line, 4u+3 = 0