Total No. of Questions—**8**] [Total No. of Printed Pages—**4+2**

Seat No.	

[4957]-1035

S.E. (Electrical/Instrumentations) (First Sem.)

EXAMINATION, 2016

ENGINEERING MATHEMATICS–III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Neat diagrams must be drawn wherever necessary.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use of electronic pocket calculator is allowed.

(*iv*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* differential equations : [8]

(*i*) $\frac{d^2y}{dx^2} + 4y = \sin x \cos 3x$

(*ii*) $(2x+1)^2 \frac{d^2y}{dx^2} - 2(2x+1)\frac{dy}{dx} - 12y = 6x$

(*iii*) $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ by the method of variation of parameters.

(*b*) Solve the differential equation by using Laplace transform method : [4]

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = e^{-2t}, \quad y = 0,\ y' = 0 \quad \text{at} \quad t = 0.$$

P.T.O.

Or

2. (*a*) An emf applied to the circuit containing a condenser C and inductance L in series at $t = 0$ is E sin pt. The current i satisfies the integro differential equation :

$$L\frac{di}{dt}+\frac{1}{C}\int i\,dt = E\sin pt$$

where $p^2 = \frac{1}{LC}$. Find the current in the circuit at any time t.

(solve by taking $i = -\frac{dq}{dt}$) [4]

(*b*) Solve any *one* of the following : [4]

(*i*) $L[t\ e^{3t}\cos 2t]$

(*ii*) $L^{-1}\left[\frac{2s+1}{(s+4)(s-6)}\right]$.

(*c*) Obtain $L[f(t)]$,

where $f(t) = ap,\ 0 < t < \frac{\pi}{p}$

$= 0\quad,\ \frac{\pi}{p} < t < \frac{2\pi}{p}$

and $f\left(t+\frac{2\pi}{p}\right) = f(t)$. [4]

3. (*a*) Find the Fourier sine transform of the function : [4]

$$F(x) = \begin{cases} x & , \quad 0 \le x \le 1 \\ 2 - x & , \quad 1 \le x \le 2 \\ 0 & , \quad x > 2 \end{cases}.$$

(*b*) Attempt any *one* : [4]

(*i*) Find z transform of :

$$F(k) = 2^k \cos(3k + 2); \quad k \ge 0 .$$

(*ii*) Find the inverse z-transform of :

$$\frac{3z^2 + 2z}{z^2 - 3z + 2}; \quad 1 < |z| < 2 .$$

(*c*) If the directional derivative of $d = axy + byz + czx$ at (1, 1, 1) has maximum magnitude 4 in a direction parallel to x-axis, find the values of a, b, c. [4]

Or

4. (*a*) Establish any *one* of the following : [4]

(*i*) $\nabla^2 \frac{(\bar{a} . \bar{b})}{r} = 0$

(*ii*) $\nabla\left(\frac{\bar{a} . \bar{r}}{r^n}\right) = \frac{\bar{a}}{r^n} - \frac{n(\bar{a} . \bar{r})}{r^{n+2}} \bar{r}$.

(*b*) For a solenoidal vector field $\bar{E}$, show that : [4]

$$\text{curl curl curl curl } \bar{E} = \nabla^4 \bar{E}$$

(*c*) Obtain $f(k)$, given that : [4]

$$12f(k+2) - 7f(k+1) + f(k) = 0 ; \quad k \ge 0, \ F(0) = 0 , \quad F(1) = 3$$

5. (*a*) Find the work done in moving a particle from the point $A(0, 1, \frac{\pi}{4})$ to point $B(\pi, 2, \frac{\pi}{2})$ in the force field : [4]

$$\bar{F} = (y\sin z - \sin x)\hat{i} + (x\sin z + 2yz)\hat{j} + (xy\cos z + y^2)\hat{k}.$$

(*b*) Evaluate : [5]

$$\iint_s (x^2y^3\,\hat{i} + z^2x^3\,\hat{j} + x^2y^3\,\hat{k}) \cdot d\bar{s},$$

where '*s*' is the surface of the sphere $x^2 + y^2 + z^2 = a^2$.

(*c*) Evaluate :

$$\iint_s (\nabla \times \bar{F}) \cdot d\bar{s},$$

where $\bar{F} = (2y + x)\hat{i} + (x - y)\hat{j} + (z - x)\hat{k}$,

and '*s*' is the surface bounded by $x = 0$, $y = 0$, $x + y + z = 1$, which is not included in XOY–plane. [4]

Or

6. (*a*) Evaluate :

$$\int_c \bar{F} \cdot d\bar{r},$$

using Green's theorem, where :

$$\bar{F} = (2x^2 - y^2)\hat{i} + (x^2 + y^2)\hat{j},$$

and '*c*' is the circle $x^2 + y^2 = 1$ above *x*-axis. [4]

(*b*) Using Gauss's Divergence Theorem, evaluate :

$$\iint_s \bar{F} \cdot \hat{n}\, ds$$

where $\bar{F} = x^3 \hat{i} + y^3 \hat{j} + z^3 \hat{k}$

and '*s*' is the surface of the sphere $x^2 + y^2 + z^2 = 25$. [5]

(*c*) Using Stokes' theorem, evaluate :

$$\int_c y dx + z dy + x dz,$$

where *c* is the curve of intersection of the surface $x^2 + y^2 + z^2 = a^2$ by the plane $x + z = a$. [4]

7. (*a*) Show that analytic function *f*(*z*) with constant modulus is constant. [4]

(*b*) Evaluate :

$$\oint_c \frac{e^{3z}}{(z - \log 2)^4}\, dz$$

where *c* is the square with vertices ± 1, ± *i*. [5]

(*c*) Find the bilinear transformation, which sends the points 1, *i*, –1 from *z*-plane into points *i*, 0, –*i* of the *w*-plane. [4]

Or

8. (*a*) Show that the function :

$$u = x^4 - 6x^2 y^2 + y^4$$

is harmonic and find the analytic function :

$$f(z) = u + iv.$$ [4]

(*b*) Evaluate :

$$\oint_c \frac{5z-2}{z(z-1)}\,dz$$

where c is $|z| = 3$. [5]

(*c*) Show that transformation :

$$w = z + \frac{1}{z} - 2i$$

maps the circle $|z| = 2$ into an ellipse. [4]