Total No. of Questions—**8**] [Total No. of Printed Pages—**6**

Seat No.	

[5152]-141

S.E. (Electrical/Instrumentation and Control) EXAMINATION, 2017

ENGG. MATHS.–III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of non-programmable electronic pocket calculator and steam tables is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $\frac{d^2y}{dx^2} - y = e^x(1+x^2)$

(*ii*) $(4x+1)^2 \frac{d^2y}{dx^2} + 2(4x+1)\frac{dy}{dx} + y = 2x+1$

(*ii*) $\frac{d^2y}{dx^2} + y = x\sin x$ (use method of variation of parameters)

P.T.O.

(*b*) Solve the differential equation by using Laplace transform method : [4]

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 12e^{-2t} \quad y(0) = 2 \quad y'(0) = 6.$$

Or

2. (*a*) An e.m.f. $E \sin pt$ is applied at $t = 0$ to a circuit containing a condenser C and inductance L in series. The current I satisfies the equation : [4]

$$L\frac{dI}{dt} + \frac{1}{C}\int I\,dt = E \sin pt$$

where $I = -\frac{dq}{dt}$ if $p^2 = \frac{1}{LC}$ and initially the current I and charge q are zero. Show that current in the circuit at any time t is $\frac{E}{2L} t \sin pt$.

(*b*) Solve any *one* of the following : [4]

(*i*) Find the Laplace transform of $f(t) = \begin{cases} \cos t & 0 < t < \pi \\ \sin t & t > \pi \end{cases}$.

(*ii*) Find the inverse Laplace transform of :

$$F(s) = \frac{s^2 + 2}{s(s^2 + 4)}.$$

(*c*) Find the inverse Laplace transform of : [4]

$$F(s) = \frac{(1 - e^{-s})^2}{(s-1)(s-2)}.$$

3. (*a*) Solve the following integral equation : [4]

$$\int_0^\infty f(x)\sin\lambda x\,dx = \begin{cases}(1-\lambda), & 0 \le \lambda \le 1\\ 0 \quad , & \lambda \ge 1\end{cases}$$

(*b*) Find the Z transform of : [4]

$$f(k) = \frac{2^k}{k!};\quad k \ge 0\,.$$

(*c*) If the directional derivative of

$$\phi = axy + byz + czx$$

at (1, 1, 1) has maximum magnitude 4 in a direction parallel to y axis, find values of a, b, c. [4]

Or

4. (*a*) Prove the following (any *one*) : [4]

(*i*) $\nabla.\left(r\,\nabla\dfrac{1}{r^n}\right) = \dfrac{n(n-2)}{r^{n+1}}$

(*ii*) $\nabla\times\left(\dfrac{\vec{a}\times\vec{r}}{r^3}\right) = -\dfrac{\vec{a}}{r^3} + \dfrac{3(\vec{a}\,.\,\vec{r})}{r^5}\vec{r}$

(*b*) Verify wheather field : [4]

$$\overline{F} = (y\sin z - \sin x)\hat{i} + (x\sin z + 2yz)\hat{j} + (xy\cos z + y^2)\hat{k}$$

is irrotational and if so, find corresponding scalar potential function ϕ.

(*c*) Solve the difference equation : [4]

$$f(k+2) + 3f(k+1) + 2f(x) = 0$$

where $f(0) = 0$ and $f(1) = 1$.

5. (*a*) Evaluate $\int_C \bar{F}.d\bar{r}$ where : [4]

$$\bar{F} = y\bar{i} + xz^3\bar{j} - y^2z\ \bar{k}$$

and C is the circle $x^2 + y^2 = 4$, $z = 1$.

(*b*) Using Stokes' theorem, evaluate $\int_C \bar{F}.d\bar{r}$, where [5]

$$\bar{F} = y^2\bar{i} + x^2\bar{j} - (x+z)\bar{k}$$

and C is the boundary of tringle with vertices (0, 0, 0), (1, 0, 0) and (1, 1, 0).

(*c*) Use divergence theorem to evaluate : [4]

$$\iint_S (x^3\bar{i} + y^3\bar{j} + z^3\bar{k}).d\bar{s}$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = 16$.

Or

6. (*a*) Evaluate $\int_C \bar{F}.d\bar{r}$ for [4]

$$\bar{F} = (2y+3)\bar{i} + xz\ \bar{j} + (yz+x)\ \bar{k}$$

along the path $x = 2t^2$, $y = t$, $z = 2$ from $t = 0$ to $t = 1$.

(*b*) Use Stokes theorem to evaluate : [5]

$$\oint_C (4y\bar{i} + 2z\bar{j} + 6y\bar{k}) . d\bar{r}$$

where C is the curve of intersection of $x^2 + y^2 + z^2 = 2z$ and $x = z - 1$.

(*c*) Prove that : [4]

$$\iiint_V \frac{dv}{r^2} = \iint_S \frac{\bar{r} . \hat{n}}{r^2} ds$$

where V is the volume bounded by closed surface S.

7. (*a*) Show that analytic function $f(z)$ with constant amplitude is constant. [5]

(*b*) Find the bilinear transformation which maps, the points $z = -1, 0, 1$ on to the points $w = 0, i, 3i$. [4]

(*c*) Evaluate : [4]

$$\frac{1}{2\pi i} \oint_C \frac{e^{2z}}{z^2 + 1} dz$$

where C is circle $|z| = 3$.

Or

8. (*a*) If $f(z) = u + iv$ is analytic, find $f(z)$ if [5]

$$u - v = (n - y)(x^2 + 4xy + y^2).$$

(*b*) Evaluate : [4]

$$\oint_C \frac{4z^2 + z}{z^2 - 1} dz$$

where C is the contour $|z-1| = \frac{1}{2}$.

(*c*) Show that X-axis in Z-plane is mapped on to the circle $|w| = 1$, under the transformation :

$$w = \frac{i-z}{i+z}.$$ [4]