Total No. of Questions—**8**] [Total No. of Printed Pages—**6**

Seat No.	

[5352]-141

S.E. (Electrical/Instrumentation & Control)

(I Sem.) EXAMINATION, 2018

ENGINEERING MATHEMATICS—III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

N.B. **:—** (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* differential equations : [8]

(*i*) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x\,e^x \sin x$

(*ii*) $r^2\frac{d^2u}{dr^2} + r\frac{du}{dr} - u = r^3$

(*iii*) $\frac{d^2y}{dx^2} + 9y = \text{cosec } 3x,$

by the method of variation of parameters.

P.T.O.

(*b*) Solve the differential equation by Laplace transform method :

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = \frac{6}{e^{2t}},$$

where $x(0) = 0$, $x'(0) = 0$. [4]

Or

2. (*a*) An electrical circuit consists of an inductance L, and condenser of capacitance C is applied with emf $E = E_0 \cos \omega t$. The charge q satisfies the differential equation :

$$\frac{d^2q}{dt^2} + \frac{1}{LC} q = \frac{E_0}{L} \cos \omega t, \quad \text{where} \quad \omega^2 = \frac{1}{LC}.$$

Show that the current at any time t is given by

$$q = q_0 \cos \omega t + \frac{i_0}{\omega} \sin \omega t + \frac{E_0}{2LW} t \sin \omega t,$$

provided $q = q_0$, $i = i_0$ at $t = 0$. [4]

(*b*) Solve any *one* of the following : [4]

(*i*) $L\left[e^{-4t} \int_0^t t \sin 3t \, dt\right]$

(*ii*) $L^{-1}\left[\log\left(\frac{s^2+1}{s^2}\right)\right]$.

(*c*) Evaluate the integral using Laplace transform : [4]

$$\int_0^\infty e^{-4t} t \sin 3t \, dt.$$

3. (*a*) Find the Fourier cosine transform of the function : [4]

$$F(x) = \begin{cases} \cos x, & 0 < x < a \\ 0, & x > a \end{cases}.$$

(*b*) Attempt any *one* : [4]

(*i*) Find z-transform of :

$$f(k) = \frac{1}{2!}(k+1)(k+2)a^k; \quad k \geq 0.$$

(*ii*) Find inverse z-transform of :

$$\frac{z^3}{(z-1)\left(z-\frac{1}{2}\right)^2}, \quad |z| > 1.$$

(*c*) Find directional derivative of :

$$\phi = 4xz^3 - 3x^2y^2z$$

at (2, –1, 2) along a line equally inclined with co-ordinate axes. [4]

Or

4. (*a*) Establish any *one* of the following : [4]

(*i*) $\nabla(\overline{r} \cdot \overline{u}) = \overline{r} \times (\nabla \times \overline{u}) + (\overline{r} \cdot \nabla)\overline{u} + \overline{u}$

(*ii*) $\nabla \times (\overline{r} \times \overline{u}) = \overline{r}(\nabla \cdot \overline{u}) - (\overline{r} \cdot \nabla)\overline{u} - 2\overline{u}.$

(*b*) Show that $\overline{F} = r^2\overline{r}$ is conservative and obtain the scalar potential associated with it. [4]

(*c*) Solve the difference equation : [4]

$$f(k + 2) + 3f(k+) + 2f(k) = 0, \quad f(0) = 0, \quad f(1) = 1.$$

5. (*a*) Find the work done by the field :

$$\bar{F} = 2xy^2\ \hat{i} + \left(2x^2 y + y\right)\hat{j} + xz^2\ \hat{k},$$

in moving a particle from the point A(0, 0, 0) to point B(2, 4, 0) along the curve $y = x^2$, $z = 0$. [4]

(*b*) Evaluate : [5]

$$\iint_S \bar{F} \cdot d\bar{S},$$

where

$$\bar{F} = \frac{\bar{r}}{r^2},$$

and 'S' is the surface of the sphere $x^2 + y^2 + z^2 = 4$.

(*c*) Evaluate : [4]

$$\iint_S \left(\nabla \times \bar{F}\right) \cdot d\bar{S},$$

where

$$\bar{F} = xy^2\ \hat{i} + y\,\hat{j} + xz^2\ \hat{k},$$

and 'S' is a rectangular surface bounded by $x = 0$, $y = 0$, $x = 1$, $y = 2$, $z = 0$.

Or

6. (*a*) Evaluate : [4]

$$\int_C \left(\sin y - y^3\right)dx + \left(xy^2 + x\cos y\right)dy,$$

using Green's theorem, where 'C' is the circle $x^2 + y^2 = a^2$.

(*b*) Using Gauss' Divergence theorem, evaluate : [5]

$$\iint_S \bar{F} \cdot d\bar{S},$$

where

$$\bar{F} = yz\,\hat{i} + xz\,\hat{j} + xy\,\hat{k},$$

and 'S' is the surface of the sphere $x^2 + y^2 + z^2 = 1$, in the first octant.

(*c*) Prove that : [4]

$$\int_C (\bar{a} \times \bar{r}) \cdot d\bar{r} = 2\iint_S \bar{a} \cdot d\bar{S},$$

where $\bar{a}$ is a constant vector.

7. (*a*) Show that analytic function $f(z)$ with constant amplitude is constant. [4]

(*b*) Evaluate : [5]

$$\oint_C \frac{z^4 - 3z^2 + 6}{(z + i)^3}\, dz$$

where C is the circle $|z| = 4$.

(*c*) Find the bilinear transformation which maps the points $z = 1, i, 2i$ on the points $\omega = -2i, 0, 1$ respectively. [4]

Or

8. (*a*) Show that $V(x, y) = -\sin x \sinh y$ is harmonic. Find harmonic conjugate of $V(x, y)$. [5]

(*b*) Evaluate : [4]

$$\oint_C \frac{2z^2 + z + 5}{\left(z - \frac{3}{2}\right)^2} \, dz$$

where C is the ellipse :

$$\frac{x^2}{4} + \frac{y^2}{9} = 1.$$

(*c*) Find the map of the straight line $y = 4x$ under the transformation : [4]

$$\omega = \frac{z-1}{z+1}.$$