Total No. of Questions : 8]

SEAT No. :

P1000

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S.E. (Electrical + SW/Instrumentation Engg.) (Semester - I)

ENGINEERING MATHEMATICS - III

(2012 Course)

Time :2 Hours] *[Max. Marks :50*

Instructions to the candidates :

1) *Attempt four questions: Q.1 or Q.2, Q.3 or Q.4, Q.5 or Q.6 and Q.7 or Q.8.*
2) *Figures to the right indicate full marks.*
3) *Assume suitable data if necessary.*
4) *Neat diagrams must be drawn wherever necessary.*
5) *Use of electronic non-programmable calculator is allowed.*

Q1) a) Solve the following (any two) : **[8]**

i) $(D^2 - 1)y = x\cos x$.

ii) $(3x+2)^2 \dfrac{d^2y}{dx^2} + 3(3x+2)\dfrac{dy}{dx} - 36y = (3x+2)^3$.

iii) $(D^2 - 1)y = \dfrac{2}{1+e^x}$, use method of variation of parameters.

b) Solve following differential equation using Laplace transform

$y'' + 4y = \sin 3t,\ y(0) = y'(0) = 0.$ **[4]**

OR

Q2) a) An inductor of 0.5 henries is connected in series with resistor of 6 ohms, a capacitor of 0.02 farads, a generator having alternative voltage of 24 sin*lot*, the differential equation of the circuit is **[4]**

$$\frac{d^2q}{dt^2} + 12\frac{dq}{dt} + 100q = 48\sin lot$$

Find q at any time t if $q = 0 = i$ at $t = 0$.

b) Solve (any one) : **[4]**

i) $L\left\{\int_0^t \dfrac{\sin t}{t} dt\right\}$

ii) $L^{-1}\left\{\dfrac{1}{(s+1)(s^2+1)}\right\}$.

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c) Find the Laplace transform of **[4]**

$f(t)=1, \quad 0 \le t < a,$

$\quad =-1 \quad a < t < 2a,$

$f(t+2a) = f(t).$

Q3) a) Find the Fourier cosine transform of **[4]**

$$f(x) = \begin{cases} x, & 0 < x < \frac{1}{2} \\ 1-x, & \frac{1}{2} < x < 1 \\ 0, & x > 1 \end{cases}$$

b) Attempt any one : **[4]**

i) Find the Z transform of $f(k) = (k+1)2^k, k \ge 0$.

ii) Find inverse Z transform of $\dfrac{z^2}{\left(z-\frac{1}{4}\right)\left(z-\frac{1}{5}\right)}, |z| > \frac{1}{4}$.

c) Find the directional derivative of $\phi = 2x^2 + 3y^2 + z^2$ at the point (2, 1, 3) along the direction of the line $2(x-2) = y-1 = z-3$. **[4]**

OR

Q4) a) Establish any one of the following : **[4]**

i) $\nabla^4 e^r = e^r + \frac{4}{r} e^r$

ii) $\bar{a}.\nabla\left[\bar{b}.\nabla\left(\frac{1}{r}\right)\right] = \dfrac{3(\bar{a}.\bar{r})(\bar{b}.\bar{r})}{r^5} - \dfrac{\bar{a}.\bar{b}}{r^3}$

b) Show that the vector field **[4]**

$\overline{F} = (x+2y+4z)i + (2x-3y-z)j + (4x-y+2z)k$ is irrotational. Find scaler potential ϕ such that $\overline{F} = \nabla\phi$.

c) Obtain f(k), given that **[4]**

$f(k+2) - 4f(k+1) + 3f(k) = 0. \ k \ge 0$

$f(0) = 0, \ f(1) = 2$

Q5) a) Find the work done in moving a particle once round the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$, $z = 0$ under the force field given by

$\overline{F} = (2x-y+5z)\bar{i} + (x+y-z)\bar{j} + (3x-2y+4z^3)\bar{k}$ **[4]**

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b) Prove that for the closed surface **[4]**

$$\iint_s \frac{\bar{r}}{r^2}.d\bar{s} = \iiint_v \frac{1}{r^2} dv$$

c) Evaluate $\iint_s \nabla \times \overline{F}.d\bar{s}$ for $\overline{F} = y\bar{i} + z\bar{j} + x\bar{k}$ where s is the surface of the paraboloid $z = 1 - x^2 - y^2,\ z \geq 0$ using Stoke's theorem. **[5]**

OR

Q6) a) Using Green's theorem evaluate **[4]**

$\oint_c [\cos y\bar{i} + x(1 - \sin y)\bar{j}].d\bar{r}$ where c is the closed curve $x^2 + y^2 = 1$, $z = 0$.

b) Evaluate $\iint \bar{r}.\hat{n}\,ds$ over the closed surface of the sphere of radius 2 with centre at origin. **[4]**

c) Evaluate $\iint (\nabla \times \overline{F}).d\bar{s}$ where $\overline{F} = (x^3 - y^3)\bar{i} - xyz\,\bar{j} + y^3\bar{k}$ and s is surface $x^2 + 4y^2 + z^2 - 2x = 4$ above the plane $x = 0$. **[5]**

Q7) a) If $v = 3x^2y - y^3$, find its harmonic conjugate u and find $f(z) = u + iv$ in terms of z. **[4]**

b) Evaluate $\int_c f(z)dz$ where $f(z) = \bar{z}$ along the closed path joining the points O(0, 0), P(1, 0), Q(1, 1). **[4]**

c) Evaluate $\int_0^{2\pi} \frac{\sin 2\theta}{5 + 4\cos\theta} d\theta$ **[5]**

OR

Q8) a) Find the analytic function $f(z) = u + iv$ where $u = r^3\cos\theta + r\sin\theta$ and express $f(z)$ in terms of z. **[4]**

b) Evaluate $\oint_c \frac{z^2 + 1}{z - 2} dz$ where c is the circle $|z - 2| = 1$. **[4]**

c) Find the bilinear transformation which maps the points 0, –1, i of the Z-plane on to points $2, \infty, \frac{1}{2}(5 + i)$ of the W-plane. **[5]**

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