Total No. of Questions—**8**] [Total No. of Printed Pages—**4+2**

Seat No.	

[4657]-531

S.E. (Electrical/Instrumentation) (I Sem.) EXAMINATION, 2014

ENGINEERING MATHEMATICS–III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Answer Q. **1** or Q. **2**, Q. **3** or Q. **4**, Q. **5** or Q. **6** and Q. **7** or Q. **8**.

(*ii*) Figures to the right indicate full marks.

(*iii*) Assume suitable data, if necessary.

(*iv*) Neat diagrams must be drawn wherever necessary.

(*v*) Use of electronic non-programmable calculator is allowed.

1. (*a*) Solve any *two* of the following : [8]

(1) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = \frac{e^{-3x}}{x^3}$

(2) $\frac{d^2y}{dx^2} - y = x \sin x$

(3) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = e^x \tan x$ (by method of variation of parameters).

P.T.O.

(*b*) Find Laplace transform of : [4]

$$f(t) = e^{-t} \sin t \, U(t-\pi).$$

Or

2. (*a*) Solve simultaneously the following equations : [4]

$$\frac{du}{dx} + v = \sin x, \quad \frac{dv}{dx} + u = \cos x.$$

(*b*) Find (any *one*) : [4]

(1) $L[(t^2 - 1)\sin 2t]$

(2) $L^{-1}\left[\frac{2(s+1)}{s^2+2s+10}\right].$

(*c*) Using Laplace transform method, solve the differential equation : [4]

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = 9$$

when $x = 0$, $y = 0$ and $\frac{dy}{dx} = 0$.

3. (*a*) Find the Fourier sine transform of the function $f(x) = e^{-x}$ and hence show that : [4]

$$\int_0^{\infty} \frac{x \sin mx}{1+x^2}\, dx = \frac{\pi}{2} e^{-m}.$$

(*b*) Find *z*-transform of *f*(*k*) where : [4]

$$f(k) = 3^k \quad , \quad k < 0$$
$$= 2^k \quad , \quad k \geq 0 .$$

(*c*) Find the directional derivative of : [4]

$$\phi = xy^2 + yz^3 \quad \text{at} \quad P(1, -1, 1)$$

towards the point Q (2, 1, –1).

Or

4. (*a*) Prove the following (any *one*) : [4]

(*i*) $\nabla . (\phi \nabla \Psi - \Psi \nabla \phi) = \phi \nabla^2 \Psi - \Psi \nabla^2 \phi$

(*ii*) $\nabla \times \left(\frac{\bar{a} \times \bar{r}}{r^3} \right) = -\frac{\bar{a}}{r^3} + \frac{3(\bar{a} . \bar{r})}{r^5} \bar{r}$.

(*b*) Find inverse *z*-transform of F(*z*) where : [4]

$$F(z) = \frac{1}{(z-5)^3}, \quad |z| > 5 .$$

(*c*) Find *f*(*k*) given that : [4]

$$f(k+1) + \frac{1}{2} f(k) = \left(\frac{1}{2} \right)^k, \quad k \geq 0, \quad f(0) = 0 .$$

5. (*a*) Evaluate : [4]

$$\int_c \bar{F} \cdot d\bar{r}$$

along the straight line joining points (0, 0, 0) and (1, 2, 3) where $\bar{F} = 3x^2 i + (2xz - y)\, j + zk$.

(*b*) Use Stokes' theorem to evaluate : [5]

$$\int_c (4y\,i + 2z\,j + 6y\,k) \cdot d\bar{r},$$

where curve '*c*' is the intersection of sphere $x^2 + y^2 + z^2 = 2z$ and $x = z - 1$.

(*c*) Prove that : [4]

$$\iint_s (\phi\nabla\psi - \psi\nabla\phi) \cdot d\bar{s} = \iiint_v (\phi\nabla^2\psi - \psi\nabla^2\phi) \cdot dv.$$

Or

6. (*a*) Find the work done in moving a particle once round the circle $x^2 + y^2 = a^2$; $z = 0$ under the force field : [4]

$$\bar{F} = (\sin y) i + x(1 + \cos y) j.$$

(*b*) Prove that : [4]

$$\int_c (\bar{a} \times \bar{r}) \cdot d\bar{r} = 2\bar{a} \cdot \iint_s d\bar{s}.$$

(*c*) Evaluate : [5]

$$\iint_s \bar{F} \cdot d\bar{s}$$

over the surface of cylinder $x^2 + y^2 = 4$, $z = 0$, $z = 3$ using divergence theorem, where $\bar{F} = 4xi - 2y^2 j + x^2 k$.

7. (*a*) If [4]

$$V = \frac{-y}{x^2 + y^2},$$

find u such that $f(z) = u + iv$ is analytic function. Write $f(z)$ in terms of z.

(*b*) Evaluate : [5]

$$\oint_c \frac{z+4}{(z^2+2z+5)} dz,$$

where c is a circle $|z - 2i| = 3/2$.

(*c*) Find the bilinear transformation, which maps points 0, –1, ∞ of z-plane onto points –1, $-(2 + i)$, $+ i$ of w-plane. [4]

Or

8. (*a*) Find the condition under which : [4]

$$u = ax^3 + bx^2y + cxy^2 + dy^3$$

is Harmonic function.

(*b*) Evaluate : [5]

$$\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}$$

using Cauchy's theorem.

(*c*) Find the image of st. line $y = x$ under the transformation $w = \frac{z-1}{z+1}$. [4]