Total No. of Questions—**8**] [Total No. of Printed Pages—**4+1**

Seat No.	

[4857]-1035

S.E. (Electrical/Instru. & Cont.) (I Sem.) EXAMINATION, 2015

ENGINEERING MATHEMATICS-III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6** and Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Use of non-programmable electronic pocket calculator is allowed.

(*iv*) Figures to the right indicate full marks.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* of the following : [8]

(*i*) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = x^3 - 4x$

(*ii*) $\frac{d^2y}{dx^2} + y = x \sin x$ (by variation of parameter)

(*iii*) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \sin(\log x^2)$

P.T.O.

(*b*) Find the Laplace transform of a function : [4]

$$f(t) = e^{-4t} . \frac{\sin 3t}{3} .$$

Or

2. (*a*) Solve : [4]

$$\frac{du}{dx} + v = \sin x, \ \frac{dv}{dx} + u = \cos x \text{ at } x = 0, \ u = 1 \text{ and } v = 0.$$

(*b*) Attempt (any *one*) : [4]

(*i*) Find the Laplace transform of a function

$$f(t) = \begin{cases} \sin \omega t \ ; \ 0 < t < \dfrac{\pi}{\omega} \\ 0 \ ; \ \dfrac{\pi}{\omega} < t < \dfrac{2\pi}{\omega} \end{cases}$$

where $f(t)$ is a periodic function.

(*ii*) Find :

$$L^{-1}\left[\log\left(\frac{s^2 - 1}{s^2}\right)\right]$$

(*c*) Solve the following differential equation by Laplace transform method : [4]

$$\frac{d^2y}{dt^2} + y = \sin 3t; \ y(0) = 0, \ y'(0) = 0 .$$

3. (*a*) Find the Fourier sine transform of the following function : [4]

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2-x, & 1 \le x \le 2 \\ 0, & x > 2 \end{cases}$$

(*b*) Find inverse Z-transform of the function : [4]

$$f(z) = \frac{z}{\left(z - \frac{1}{4}\right)\left(z - \frac{1}{5}\right)} \quad \frac{1}{5} < |z| < \frac{1}{4}$$

(*c*) If directional derivative of $\phi = axy + byz + cxz$ at (1, 1, 1) has maximum magnitude 4 in a direction parallel to *x*-axis, find the values *a*, *b*, *c*. [4]

Or

4. (*a*) Prove the following (any *one*) : [4]

(*i*) $$\nabla \times \left(\frac{\bar{a} \times \bar{r}}{r^n}\right) = \frac{2-n}{r^n}\,\bar{a} + \frac{n(\bar{a}\,.\,\bar{r})\bar{r}}{r^{n+2}}$$

(*ii*) $$\nabla^4 (r^2 \log r) = \frac{6}{r^2}$$

(*b*) Find the angle between tangents at $t = 1$ and $t = 2$ to the curve $x = t^2 + 1,\ y = 4t - 3,\ z = 2t^2 - 6t$. [4]

(*c*) Solve the difference equation : [4]

$$12f(k+2) - 7f(k+1) + f(k) = 0,\ k \ge 0,\ f(0) = 0,\ f(1) = 3.$$

5. (*a*) Evaluate :

$$\int_C \overline{F} \, . \, d\overline{r}$$

for $\overline{F} = (2y + 3)\overline{i} + xz\overline{j} + (yz - x)\overline{k}$

along the straight line joining (0, 0, 0) to (3, 1, 1). [4]

(*b*) Using Stokes' theorem evaluate :

$$\int_C (x + y)dx + (2x - z)dy + (y + z)dx$$

where C is given by $x^2 + y^2 + z^2 - 2ax - 2ay = 0$, $x + y = 2a$. [5]

(*c*) Evaluate $\iint_S (4xz\,\overline{i} - y^2\overline{j} + yz\,\overline{k}) . d\overline{s}$ over the cube bounded by the planes $x = 0$, $x = 2$, $y = 0$, $y = 2$, $z = 0$, $z = 2$. [4]

Or

6. (*a*) Using Green's lemma, show that the area bounded by a simple closed curve C is given by $\frac{1}{2}\int x\,dy - y\,dx$. Hence find the area of ellipse $x = a \cos \theta$, $y = b \sin \theta$. [4]

(*b*) Evaluate $\iint \overline{r} \, . \, \hat{n}$ over the surface of a sphere of radius 1, with centre at origin. [5]

(*c*) Maxwell's electromagnetic equations are :

$$\nabla \, . \, \overline{B} = 0, \; \nabla \times \overline{\varepsilon} = -\frac{\partial \overline{B}}{\partial t}.$$

Given $\overline{B} = \nabla \times \overline{A}$, deduce that $\overline{\varepsilon} + \frac{\partial \overline{A}}{\partial t} = -\nabla v$, where v is a scalar point function. [4]

7. (*a*) If

$$u = \frac{1}{2}\log(x^2 + y^2)$$

find v such that $f(z) = u + iv$ is analytic. Determine $f(z)$ in terms of z. [4]

(*b*) Evaluate :

$$\int_C \frac{z^2 + 1}{z - 2} dz$$

where

(*i*) C is the circle $|z - 2| = 1$

(*ii*) C is the circle $|z| = 1$ [5]

(*c*) Find the bilinear transformation which maps the points $z = -1, 0, 1$ onto the points $w = 0, i, 3i$. [4]

Or

8. (*a*) Find the conditions under which

$$u = ax^3 + bx^2y + cxy^2 + dy^3$$

is harmonic. [4]

(*b*) Show that the transformation $w = z + \frac{1}{z} - 2i$ maps the circle $|z| = 2$ into an ellipse. Find its semi-major and minor axes. [5]

(*c*) Evaluate :

$$\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z - 1)(z - 2)} dz$$

where

C is the circle $|z| = 3$. [4]