Total No. of Questions—**8**] [Total No. of Printed Pages—**6**

Seat No.	

[5057]-231

S.E. (Electrical/Instrumentation) (First Semester)

EXAMINATION, 2016

ENGINEERING MATHEMATICS—III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

N.B. **:—** (*i*) Attempt Q. No. **1** *or* Q. No. **2,** Q. No. **3** *or* Q. No. **4,** Q. No. **5** *or* Q. No. **6,** Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of non-programmable electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = xe^{-x/2} \cos x$

(*ii*) $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \log x$

(*iii*) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x$ (use method of variation of parameters).

P.T.O.

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(*b*) Solve the differential equation by using Laplace transform method : [4]

$$\frac{dy}{dt} + 3y(t) + 2\int_0^t y(t)\,dt = t$$

given $y(0) = 0$.

Or

2. (*a*) An inductor of 0.25 henry is connected in series with a capacitor of 0.04 farads and a generator having alternative voltage given by 12 sin $10t$. Find the charge and current at any time t. [4]

(*b*) Solve any *one* of the following : [4]

(*i*) Find the Laplace transform of :

$$\frac{d}{dt}\left(\frac{1 - \cos t}{t}\right).$$

(*ii*) Find the inverse Laplace transform of :

$$\frac{S}{S^4 + S^2 + 1}.$$

(*c*) Find the Laplace transform of : [4]

$$f(t) = (1 + 2t + 3t^2)\, u(t - 2) + \sin 2t\; \delta\left(t - \frac{\pi}{4}\right).$$

3. (*a*) Find the Fourier transform of : [4]

$$f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}.$$

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(*b*) Find :

$$z^{-1}\left\{\frac{1}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}\right\}$$

for $|z| > \frac{1}{2}$. [4]

(*c*) Find the constants *a* and *b*, so that the surface

$$ax^2 - byz = (a + 2)x$$

will be orthogonal to the surface

$$4x^2y + z^3 = 4$$

at the point (1, –1, 2). [4]

Or

4. (*a*) Show that the vector field $f(r)\vec{r}$ is always irrotational and determine $f(r)$ such that the field is solenoidal also. [4]

(*b*) Prove the following (any *one*) : [4]

(*i*) $\nabla^2\left(\nabla \cdot \frac{\vec{r}}{r^2}\right) = \frac{2}{r^4}$

(*ii*) $\nabla \times \left(\vec{a} \times \nabla \frac{1}{r}\right) = \frac{\vec{a}}{r^3} - \frac{3\left(\vec{a} \cdot \vec{r}\right)\vec{r}}{r^5}$.

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(*c*) Solve the difference equation :

$$f(k + 2) - 3f(k + 1) + 2f(k) = 1$$

where $f(0) = 0$, $f(1) = 3$. [4]

5. (*a*) Evaluate :

$$\int_{C} \overline{F} \cdot d\overline{r}$$

for

$$\overline{F} = (2y + 3)\overline{i} + xz\overline{j} + (yz - x)\overline{k}$$

along a straight line joining (0, 0, 0) to (3, 1, 1). [4]

(*b*) Evaluate :

$$\iint_{S} (\nabla \times \overline{F}) \cdot \hat{n} \, dS$$

where S is the curved surface of the paraboloid

$$x^2 + y^2 = 2z$$

bounded by the plane $z = 2$ where

$$\overline{F} = 3(x - y)\overline{i} + 2xz\overline{j} + xy\overline{k}.$$ [5]

(*c*) Evaluate :

$$\iint \overline{r} \cdot \hat{n} \, dS$$

over the surface of sphere of radius 2 with centre at origin. [4]

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Or

6. (*a*) Using Green's theorem evaluate : [4]

$$\int_C \left[\cos y\,\bar{i} + x(1-\sin y)\bar{j}\right]\cdot d\bar{r}$$

where C is the closed curve

$$x^2 + y^2 = 1, \; z = 0.$$

(*b*) Prove that :

$$\int_C (\bar{a}\times\bar{r})\cdot d\bar{r} = 2\bar{a}\cdot\iint_S d\bar{S}$$

where C is open surface bounded by closed curve C. [4]

(*c*) Evaluate :

$$\iint_S \bar{F}\cdot\hat{n}\, dS$$

where

$$\bar{F} = (2x+3z)\bar{i} - (xz+y)\bar{j} + \left(y^2+2z\right)\bar{k}$$

and S is surface of sphere with radius 3. [5]

7. (*a*) If

$$u = \log(x^2 + y^2),$$

find v such that

$$f(z) = u + iv$$

is analytic. Determine $f(z)$ in terms of z. [5]

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(*b*) Evaluate :

$$\oint_C \frac{dz}{z^2}$$

where C is the circle $|z| = 1$. [4]

(*c*) Find the bilinear transformation which maps the points $0, -1, i$ of z-plane on to the points $2, \infty, \frac{1}{2}(5 + i)$ of the w-plane. [4]

Or

8. (*a*) Show that the map of straight line parallel to x-axis is family of ellipses under the transformation

$$w = \sinh (z).$$ [4]

(*b*) Evaluate :

$$\oint_C \frac{z+2}{z^2+1} dz$$

where C is the circle $|z - i| = \frac{1}{2}$. [4]

(*c*) Find analytic function

$$f(z) = u + iv$$

where

$$u = r^3 \cos 3\theta + r \sin \theta.$$ [5]

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