Total No. of Questions—**8**]

[Total No. of Printed Pages—**4**

Seat No.	

[5252]-141

S.E. (Electrical/ Instrumentation) (First Semester)

EXAMINATION, 2017

ENGINEERING MATHEMATICS-III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Attempt Q. **1** *or* **2**, Q. **3** *or* **4**, Q. **5** *or* **6**, Q. **7** *or* **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of non-programmable electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $\frac{d^2y}{dx^2} + y = \sin 3x \ \cos 2x$

(*ii*) $(2x+3)^2 \frac{d^2y}{dx^2} + (2x+3)\frac{dy}{dx} - 2y = 24x^2$

(*iii*) $\frac{d^2y}{dx^2} + 9y = \frac{1}{1+\sin 3x}$ (Use method of variation of parameters)

(*b*) Solve the differential equation by using Laplace transfrom method : $\frac{d^2y}{dt^2} + 8\frac{dy}{dt} = 8\cos t$ given $y(\pi) = -1$, $y'(0) = -1$. [4]

P.T.O.

Or

2. (*a*) An inductor of 0.5 henry is connected in series with resistor of 6 ohms, a capacitor of 0.02 farad. A generator having alternative voltage of 24 sin10t. Find q at any time t if $q = 0$, I = 0 at $t = 0$.

(*b*) Solve any *one* of the following : [4]

(*i*) Find the Laplace transform of $f(t) = \dfrac{1-e^{-bt}}{t}$

(*ii*) Find the inverse Laplace transform of

$$F(S) = \frac{1}{S}\log\left(1+\frac{1}{S^2}\right)$$ [4]

(*c*) Using unit step function find the Laplace transform of the function $f(t) = K(n-1)$ for $(n-1)\,T < t < nT$, $n = 1, 2, 3....$ where T is period. [4]

3. (*a*) Represent the function in the Fourier integral form for $f(x) = e^{-|x|};\ -\infty < x < \infty$ [4]

(*b*) Find $z^{-1}\left\{\dfrac{z^2}{z^2+1}\right\}$, by using inversion integral method. [4]

(*c*) Find the directional derivative of $\phi = e^{2x}\cos yz$ at (0,0,0) in the direction of tangent to the curve $x = a\sin t$, $y = a\cos t$, $z = at$ at $t = \pi/4$. [4]

Or

4. (*a*) Prove the following (any *one*) : [4]

(*i*) $\nabla^4\,(r^2\log r) = \dfrac{6}{r^2}$

(*ii*) $\nabla.\left(r\nabla\dfrac{1}{r^5}\right) = \dfrac{15}{r^6}$.

(*b*) Shor that $\vec{F} = \frac{1}{r}[r^2\vec{a} + (\vec{a}.\vec{r})\vec{r}]$ is irrotational. Hence find scalar potential function ϕ. [4]

(*c*) Solve the difference equation : [4]

$$y_k - \frac{5}{6}y_{k-1} + \frac{1}{6}y_{k-2} = \left(\frac{1}{2}\right)^k ; k \geq 0$$

5. (*a*) Using Green's theorem, evaluate $\int_c [\cos y\bar{i} + x(1-\sin y)\bar{j}].d\bar{r}$ where c is the closed curve $x^2 + y^2 = 1$, $z = 0$. [4]

(*b*) Evaluate $\iint_S (\nabla \times \bar{F}).\hat{n}\, ds$ for $\bar{F} = x^2\bar{i} + y^2z\bar{j} + xy\bar{k}$ for the plane surface S bounded by $x = 0$, $y = 0$, $x = 2$, $y = 2$, $z = 0$. [5]

(*c*) Evaluate : $\iint_s [3x\, dy\, dz - 2y\, dz\, dx + 2z\, dx\, dy]$ over the surface of sphere of radius a. [4]

Or

6. (*a*) Find the work done by $\bar{F} = x^2\bar{i} + yz\bar{j} + z\bar{k}$ in moving a particle along the straight line segment from (1, 2, 2) to (3, 4, 4). [4]

(*b*) Evaluate $\iint_S \bar{F}.\hat{n}\, dS$ where $\bar{F} = x^3\bar{i} + y^3\bar{j} + z^3\bar{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ [5]

(*c*) Evaluate $\iint_S (\nabla \times \bar{F}).d\bar{S}$ for $\bar{F} = y\bar{i} + z\bar{j} + x\bar{k}$, where S is the surface of paraboloid $z = 1 - x^2 - y^2$, $z \geq 0$. [4]

7. (*a*) If $f(z)$ is analytic function, then prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. [5]

(*b*) Find the bilinear transformation, which maps the points 1, i, -1 from Z-plane onto the points i, 0, $-i$ of the w-plane. [4]

(*c*) Evaluate $\oint_c \frac{e^{2z}}{(z+1)^4}\,dz$ where c is the circle $|z| = 3$. [4]

Or

8. (*a*) If $u = x^4 - 6x^2y^2 + y^4$, find its harmonic conjugate v, find $f(z) = u + iv$ in terms of z. [5]

(*b*) Find the map of straight line $y = x$ under the transformation $w = \frac{z-1}{z+1}$. [4]

(*c*) Evaluate $\oint_c \frac{e^{2z}}{(z-1)(z-2)}\,dz$ where c is the circle $|z| = 3$. [4]