Total No. of Questions : 8]

SEAT No. :

P2021

[Total No. of Pages : 4

F.E.

ENGINEERING MATHEMATICS - I

(2012 Pattern)

Time : 2 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt four questions : Q.no. 1 or 2, Q.no. 3 or 4, Q.no. 5 or 6 and Q.no. 7 or 8.*
2) *Figures to the right indicate full marks.*
3) *Assume suitable data if necessary.*
4) *Neat diagram must be drawn wherever necessary.*
5) *Use of electronic non-programmable calculator is allowed.*

Q1) a) Show that the system of equations. **[4]**

$3x + 4y + 5z = \alpha$

$4x + 5y + 6z = \beta$

$5x + 6y + 7z = \gamma$

is consistent only of α, β and γ are in arithmetic progression.

b) Verify cayley - Hamilton theorm for A and hence find A^{-1}. **[4]**

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$$

c) If $\alpha = 1 + i$, $\beta = 1 - i$ and $\cot Q = x + 1$ then prove that

$$\frac{(x+\alpha)^n - (x+\beta)^n}{\alpha - \beta} = \sin nQ \ \operatorname{cosec}^n Q$$ **[4]**

OR

Q2) a) Examine whether the following vectors are linearly dependent or independent. If dependent, find the relation between them. **[4]**

$X_1 = [1, 2, 3]$, $X_2 = [3, -2, 1]$, $X_3 = [1, -6, -5]$

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b) Show that **[4]**

$$\log\left[\frac{\sin(x+iy)}{\sin(x-iy)}\right]=2i\tan^{-1}(\cot x\tan hy).$$

c) Prove that $\tan^{-1}\left[i\left(\frac{x-a}{x+a}\right)\right]=-\frac{i}{2}\log\left(\frac{a}{x}\right).$ **[4]**

Q3) a) Test the convergence of the series (Any one) **[4]**

i) $\frac{2}{1}+\frac{3}{8}+\frac{4}{27}+\frac{5}{64}+\text{............}+\frac{n+1}{n^3}+\text{..........}$

ii) $1-\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}-\frac{1}{4\sqrt{4}}+\text{............}$

b) Expand x^3+7x^2+x-6 in powers of $(x-3)$ **[4]**

c) If $y=\log\left(x+\sqrt{x^2+1}\right)$, prove that

$$(1+x^2)y_{n+2}+(2n+1)x\,y_{n+1}+n^2y_n=0.$$ **[4]**

OR

Q4) a) Solve any one **[4]**

i) Evaluate $\lim_{x\to\pi/2}(\cos x)^{\cos x}$.

ii) If $\lim_{x\to 0}\frac{\sin 2x+\rho\sin x}{x^3}$ is finite, find the value of ρ and hence evaluate the limit.

b) Prove that $\log(1+\tan x)=x-\frac{x^2}{2}+\frac{2x^3}{3}\text{.............}$ **[4]**

c) Find n[th] derivative of $y=\frac{x}{(x-1)(x-2)(x-3)}$. **[4]**

Q5) Solve any two of the following :

a) Find the value of n such that $u = x^n (3\cos^2 y - 1)$ satisfies the partial differential equation. **[6]**

$$\frac{\partial}{\partial x}\left(x^2 \frac{\partial u}{\partial x}\right) + \frac{1}{\sin y}\frac{\partial}{\partial y}\left(\sin y \frac{\partial u}{\partial y}\right) = 0$$

b) If $x = r\cos\theta,\ \ y = r\sin\theta$

then prove that **[6]**

i) $\left(\frac{\partial y}{\partial r}\right)_x \left(\frac{\partial y}{\partial r}\right)_\theta = 1$

ii) $\left(\frac{\partial x}{\partial \theta}\right)_r = r^2 \left(\frac{\partial \theta}{\partial x}\right)_y$

c) If $u = x^8 f\left(\frac{y}{x}\right) = \frac{1}{y^8}\phi\left(\frac{x}{y}\right)$, **[7]**

then prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 64u$$

OR

Q6) Solve any two of the following :

a) If $u = \cos^{-1}\left[\frac{x^3 y^2 + 4y^3 x^2}{\sqrt{x^4 + 6y^4}}\right]$ **[7]**

find the value of

i) $x\,\frac{\partial u}{\partial x} + y\,\frac{\partial u}{\partial y}$

ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\,\frac{\partial^2 u}{\partial x \partial y} + y^2\,\frac{\partial^2 u}{\partial y^2}$

b) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$.

prove that **[6]**

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$$

c) If $f(x, y) = 0$, $\phi(x, z) = 0$ **[6]**

then prove that

$$\frac{\partial \phi}{\partial x}\frac{\partial f}{\partial y}\frac{dy}{dz} = \frac{\partial f}{\partial x}\frac{\partial \phi}{\partial z}$$

Q7) a) If $x = e^v \sec u$, $y = e^v \tan u$, find $\frac{\partial(u,v)}{\partial(x,y)}$. **[4]**

b) Examine for functional dependence for **[4]**

$$u = \frac{x}{y-z}, \quad v = \frac{y}{z-x}, \quad w = \frac{z}{x-y}$$

c) Find extreme values of $f(x, y) = x^3 + y^3 - 3axy$, $a > 0$ **[5]**

OR

Q8) a) If $x = \cos\theta - r\sin\theta$, $y = \sin\theta + r\cos\theta$ find $\frac{\partial r}{\partial x}$. **[4]**

b) The resonant frequency in a series electrical circuit is given by $f = \frac{1}{2\pi\sqrt{LC}}$. If the measurement in L and C are in error by 2% and –1% respectively. Find the percentage error in f. **[4]**

c) Use Lagrange's method to find stationary value of $u = \frac{x^2}{a^3} + \frac{y^2}{b^3} + \frac{z^2}{c^3}$ where $x + y + z = 1$. **[5]**

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