Total No. of Questions—8] [Total No. of Printed Pages—4+2

Seat No.	

[4756]-101

F.E. (I Sem.) EXAMINATION, 2015

ENGINEERING MATHEMATICS—I

(2012 COURSE)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Attempt *All* questions.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of logarithmic tables, electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (A) Find the eigen values and eigen vector corresponding to minimum eigen value for the matrix : [4]

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}.$$

P.T.O.

(B) Determine the value for λ for which the equations :

$$3x_1 + 2x_2 + 4x_3 = 3,$$

$$x_1 + x_2 + x_3 = \lambda,$$

$$5x_1 + 4x_2 + 6x_3 = 15,$$

are consistent. Find also the corresponding solution. [4]

(C) If z_1, z_2, z_3 are the vertices of an isosceles triangle right angled at z_2, prove that : [4]

$$z_1^2 + z_3^2 + 2z_2^2 = 2z_2\left(z_1 + z_3\right).$$

Or

2. (A) If [4]

$$2\cos\theta = x + \frac{1}{x},$$

prove that :

$$2\cos r\theta = x^r + \frac{1}{x^r}.$$

(B) If $Y = \log\tan x$, prove that : [4]

(*i*) $\sinh ny = \frac{1}{2}\left(\tan^n x - \cot^n x\right),$

(*ii*) $2\cosh ny \operatorname{cosec} 2x = \cosh\left(n+1\right)y + \cosh\left(n-1\right)y.$

(C) Examine for linear dependance or independance for the given vectors and if dependance, find the relation between them : [4]

$$X_1 = (1, -1, 2, 2),$$
$$X_2 = (2, -3, 4, -1),$$
$$X_3 = (-1, 2, -2, 3).$$

3. (A) Test convergence of the series (any *one*) : [4]

(*i*) $\sum_{n=1}^{\infty} \left(\frac{2n+1}{3n+4}\right)5^n$

(*ii*) $\frac{1}{1+\sqrt{2}} + \frac{2}{1+2\sqrt{3}} + \frac{3}{1+3\sqrt{4}} + \ldots\ldots\ldots\ldots$

(B) Prove that : [4]

$$\sin x \cosh x = x + \frac{1}{3}x^3 - \frac{1}{30}x^5 - \ldots\ldots\ldots\ldots$$

(C) Find *n*th derivative of : [4]

$$e^{2x} \sinh 3x \cos 4x.$$

Or

4. (A) Solve any *one* : [4]

(*i*) $\lim_{x \to a} (x-a)^{(x-a)}$

(*ii*) $\lim_{x \to \pi/2} (\sec x - \tan x).$

(B) Using Taylor's theorem, expand :

$$2x^3 + 3x^2 - 8x + 7$$

in powers of $x - 2$. [4]

(C) If $y = e^{\tan^{-1} x}$, then show that : [4]

$$\left(1 + x^2\right) y_{n+1} + (2nx - 1) y_n + n(n-1) y_{n-1} = 0.$$

5. Solve any *two* :

(A) Find the value of n for which : [6]

$$z = A\ e^{-gx} \sin(nt - gx),$$

satisfies the partial differential equation :

$$\frac{\partial z}{\partial t} = \frac{\partial^2 z}{\partial x^2}.$$

(B) If

$$u = \frac{x^4 + y^4}{x^2 y^2} + x^6 \tan^{-1}\left[\frac{x^2 + y^2}{x^2 + 2xy}\right],$$

find the value of :

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$$

at $x = 1$, $y = 2$. [7]

(C) If $z = f(u, v)$ and [6]

$$u = \log\left(x^2 + y^2\right), v = \frac{y}{x},$$

show that :

$$x \frac{\partial z}{\partial y} - y \frac{\partial z}{\partial x} = \left(1 + v^2\right) \frac{\partial z}{\partial v}.$$

Or

6. Solve any *two* :

(A) If [6]

$$x = \frac{\cos\theta}{r}, \; y = \frac{\sin\theta}{r},$$

find the value of :

$$\left(\frac{\partial x}{\partial r}\right)_\theta \left(\frac{\partial r}{\partial x}\right)_y + \left(\frac{\partial y}{\partial r}\right)_\theta \left(\frac{\partial r}{\partial y}\right)_x.$$

(B) If [7]

$$u = \sin^{-1}\sqrt{\frac{x^2 + y^2}{x + y}},$$

show that :

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{1}{4} \tan u \left[\tan^2 u - 1\right].$$

(C) If $z = f(u, v)$ and $u = x\cos t - y\sin t$, $v = x\sin t + y\cos t$, where t is a constant, prove that : [6]

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = u\frac{\partial z}{\partial u} + v\frac{\partial z}{\partial v}.$$

7. (A) If $u^3 + v^3 = x + y$, $u^2 + v^2 = x^3 + y^3$, find $\frac{\partial(u, v)}{\partial(x, y)}$. [4]

(B) Determine whether the following functions are functionally dependent. If functionally dependent, find the relation between them : [4]

$$u = \sin x + \sin y, \; v = \sin(x + y).$$

(C) Examine maxima and minima of the following function and find their extreme values : [5]

$$\left(x^2 + y^2 + 6x + 12\right).$$

Or

8. (A) If $x = u + v$, $y = v^2 + w^2$, $z = w^3 + u^3$, show that : [4]

$$\frac{\partial u}{\partial x} = \frac{vw}{vw + u^2}.$$

(B) If $e^z = \sec x \cos y$ and errors of magnitude h and $-h$ are made in estimating x and y, where x and y are found to be $\frac{\pi}{3}$ and $\frac{\pi}{6}$ respectively, find the corresponding error in z. [5]

(C) Find the minimum distance from origin to the plane : [4]

$$3x + 2y + z = 12.$$