Total No. of Questions—**8**] [Total No. of Printed Pages—**4+2**

Seat No.	

[4956]-101

F.E. (Common) EXAMINATION, 2016

ENGINEERING MATHEMATICS—I

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of non-programmable electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Show that the system

$$3x + 4y + 5z = \alpha,$$
$$4x + 5y + 6z = \beta,$$
$$5x + 6y + 7z = \gamma$$

is consistent only when $2\beta = \alpha + \gamma$. [4]

(*b*) Find eigen values and eigen vector corresponding to lowest eigen value for the matrix. [4]

$$A = \begin{bmatrix} 0 & 2 & 0 \\ 3 & -2 & 3 \\ 0 & 3 & 0 \end{bmatrix}$$

P.T.O.

(*c*) If [4]

$$\operatorname{cosec}\left(\frac{\pi}{4} + ix\right) = u + iv$$

where x, u, v are real, show that

$$(u^2 + v^2)^2 = 2(u^2 - v^2)$$

Or

2. (*a*) Examine for linear dependence of vectors [4]

(1, 2, –1, 0), (1, 3, 1, 2), (4, 2, 1, 0), (6, 1, 0, 1).

(*b*) Show that [4]

$$\left|\frac{z}{|z|} - 1\right| \le |\arg z|$$

(*c*) Prove that i^i is wholly real and find its principal value. [4]

3. (*a*) Test the convergence of the series (any *one*) [4]

(*i*) $1 + \frac{3}{2!} + \frac{3^2}{3!} + \frac{3^3}{4!} + \frac{3^4}{5!} + \ldots\ldots\ldots\ldots\ldots\ldots .$

(*ii*) $\sum \frac{1}{n} \cos\left(\frac{1}{n}\right)$

(*b*) Expand

$$\sin^{-1}\left(\frac{2x}{1 + x^2}\right)$$

in ascending powers of x [4]

(*c*) Find the n^{th} derivative of [4]

$$\frac{1}{(x-1)^2(x-2)}.$$

Or

4. (*a*) Solve any *one* : [4]

(*i*) $\lim_{x \to 0} (1 + \tan x)^{\cot(x)}$

(*ii*) Find the values of a and b such that

$$\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1.$$

(*b*) Using Taylor's theorem expand [4]

$$x^4 - 3x^3 + 2x^2 - x + 1$$

in powers of $(x - 3)$

(*c*) If

$$y = (\sin^{-1} x)^2,$$

prove that [4]

$$(1 - x^2)y_{n+2} - (2n + 1)x\, y_{n+1} - n^2 y_n = 0.$$

5. Solve any *two* questions :

(*a*) If [6]

$$u = \log(x^3 + y^3 + z^3 - 3xyz),$$

prove that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x + y + z)^2}.$$

(*b*) If [7]

$$z = x^8 f\left(\frac{y}{x}\right) + y^{-8}\ \phi\left(\frac{x}{y}\right),$$

prove that

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = 64z + 8y^{-8}Q\left(\frac{x}{y}\right) - 8x^8 f\left(\frac{y}{x}\right)$$

(*c*) If [6]

$$u = f\ (2x - 3y,\ 3y - 4z,\ 4z - 2x),$$

then find value of

$$\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z}.$$

Or

6. Solve any *two* questions :

(*a*) If [6]

$$ux + vy = 0,\ \frac{u}{x} + \frac{v}{y} = 1$$

prove that

$$\left(\frac{\partial u}{\partial x}\right)_y - \left(\frac{\partial v}{\partial y}\right)_x = \frac{y^2 + x^2}{y^2 - x^2}.$$

(*b*) If [6]

$$u = \sec^{-1}\left[\frac{x + y}{\sqrt{x} + \sqrt{y}}\right],$$

prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{1}{4}\ \cot u\left[3 + \cot^2 u\right].$$

(*c*) If

$$\phi = f(x, y, z), \quad x = \sqrt{vw}, \; y = \sqrt{uw}, \; z = \sqrt{uv},$$

prove that :

$$x\frac{\partial \phi}{\partial x} + y\frac{\partial \phi}{\partial y} + z\frac{\partial \phi}{\partial z} = u\frac{\partial \phi}{\partial u} + v\frac{\partial \phi}{\partial v} + w\frac{\partial \phi}{\partial w}.$$ [7]

7. (*a*) If

$$x = uv \text{ and } y = \frac{u + v}{u - v},$$

find $\frac{\partial(u, v)}{\partial(x, y)}$. [4]

(*b*) Find the percentage error in the area of an ellipse when an error of 1% each is made in measuring its semimajor and seminor axes. [4]

(*c*) Find the extreme values of : [5]

$$f(x, y) = xy(a - x - y).$$

Or

8. (*a*) Examine whether the following functions are functionally dependent, if so find the relation betwen them [4]

$$u = \frac{x + y}{1 - xy}, \quad v = \tan^{-1} x + \tan^{-1} y.$$

(*b*) A balloon is in the form of right circular cylinder of radius 1.5 m and length 4 m and is surrounded by hemispherical ends. If the radius is increased by 0.01 m and the length by 0.05 m, find the percentage change in the volume of a balloon. [4]

(*c*) As the dimensions of a triangle ABC are varied, show that the maximum value of cosA cosB cosC is obtained when the triangle is equilateral. [5]