Seat
No.

F.E. (Semester – I) Examination, 2014
ENGG. MATHEMATICS – I
(2012 Pattern)

Time : 2 Hours   Max. Marks : 50

Instructions *: 1) Attempt Q.* ***1*** *or Q.* ***2****, Q.* ***3*** *or Q.* ***4****, Q.* ***5*** *or Q.* ***6****, Q.* ***7*** *or Q.* ***8****.*
2) Neat diagrams must be drawn ***wherever*** *necessary.*
3) Black figures to the ***right*** *indicate full marks.*
4) Use of logarithmic tables slide rule, Mollier charts, electronic pocket calculator and steam tables is ***allowed****.*
5) Assume suitable data, if ***necessary****.*

1. A) Examine for consistency the system of equations

   $x - y - z = 2$
   $x + 2y + z = 2$
   $4x - 7y - 5z = 2$

   and solve it if consistent.   **4**

   B) Examine whether the following vectors are linearly dependent or independent. If dependent, find the relation between them.

   $X_1 = [1 \ -1 \ 2 \ ] \ X_2 = [\ 2 \ 3 \ 5 \ ] \ X_3 = [3 \ 2 \ 1]$ **4**

   C) If cosec (x + iy) = u + iv , prove that   **4**

   i) $\dfrac{u^2}{\sin^2 x} - \dfrac{v^2}{\cos^2 x} = (u^2 + v^2)^2$

   ii) $\dfrac{u^2}{\cosh^2 y} + \dfrac{v^2}{\sinh^2 y} = (u^2 + v^2)^2$

                            OR

2. A) A square lies above real axis in Argand diagram and two of its adjacent vertices are the origin and the point 2 + 3i. Find the complex numbers representing other two vertices.   **4**

   B) If arg $(z + 1) = \dfrac{\pi}{6}$ and arg $(z - 1) = \dfrac{2\pi}{3}$ then find z.   **4**

   C) Find the eigen values and eigen vectors of following matrix.   **4**

   $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

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3. a) Test convergence of the series (**any one**).   **4**

   i) $\dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{9}} + \dfrac{1}{\sqrt{28}} + \dfrac{1}{\sqrt{65}} + ...$

   ii) $1 - \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{3}} - \dfrac{1}{\sqrt{4}} + ...$

   b) Prove that $\log(1 + \sin x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{6} - \dfrac{x^4}{12} + ...$ **4**

   c) Find n^{th} derivative of $\dfrac{x^2}{(x-1)(x-2)}$.   **4**

                              OR

4. a) Solve **any one**.   **4**

   i) Evaluate $\displaystyle\lim_{x \to o} \log_{\tan x} \tan 4x$.

   ii) Find the values of a and b if $\displaystyle\lim_{n \to o} \left[x^{-3} \sin x + ax^{-2} + b \right] = 0$.

   b) Using Taylor's theorem expand $49 + 69x + 42x^2 + 11x^3 + x^4$ in powers of (x+2).   **4**

   c) If $y = \sin \log(x^2 + 2x + 1)$, then prove that

   $(x+1)^2 y_{n+2} + (2n+1)(x+1) y_{n+1} + (n^2+4) y_{n=0}$ **4**

5. Solve **any two** of the following.

   a) If $u = \log(x^3 + y^3 - x^2 y - xy^2)$ then prove that $x^2 u_{xx} + 2_{xy} u_{xy} + y^2 u_{yy} = -3$.   **7**

   b) If $x = u + v + w$, $y = uv + vw + wu$, $z = uvw$ and ϕ is a function of x, y, z then prove that

   $u\dfrac{\partial \phi}{\partial u} + v\dfrac{\partial \phi}{\partial v} + w\dfrac{\partial \phi}{\partial w} = x\dfrac{\partial \phi}{\partial x} + 2y\dfrac{\partial \phi}{\partial y} + 3z\dfrac{\partial \phi}{\partial z}$.   **6**

   c) If ux+vy = 0 and $\dfrac{u}{x} + \dfrac{v}{y} = 1$, then prove that $\left(\dfrac{\partial u}{\partial x}\right)y - \left(\dfrac{\partial v}{\partial y}\right)x = \dfrac{x^2+y^2}{y^2-x^2}$.   **6**

                              OR

6. Solve **any two** of the following.

   a) If $u = \cos\left(\dfrac{xy}{x^2+y^2}\right) + \sqrt{x^2+y^2} + \dfrac{xy^2}{x+y}$, then find the value of $xu_x + yu_y$ at (3 ,4).   **7**

   b) If $x = \dfrac{\cos\theta}{u}$, $y = \dfrac{\sin\theta}{u}$, then prove that $u\dfrac{\partial z}{\partial u} - \dfrac{\partial z}{\partial \theta} = (y-x)\dfrac{\partial z}{\partial x} - (y-x)\dfrac{\partial z}{\partial y}$.   **6**

   c) If $u = (x^2 - y^2) f(xy)$, then show that $u_{xx} - u_{yy} = (x^4 - y^4) f''(xy)$.   **6**

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7. a) If $x = r \sin\theta \cos\phi$, $y = r \sin\theta \sin\phi$, $z = r \cos\theta$ find $\dfrac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$. **4**

   b) Examine for functional dependence $u = \sin^{-1} x + \sin^{-1} y$, $v = x\sqrt{1-y^2} + y\sqrt{1-x^2}$ if dependent find the relation between them. **4**

   c) The area of a triangle ABC is calculated from the formula $\Delta = \dfrac{1}{2} bc \sin A$. Errors of 1%, 2% and 3% respectively are made in measuring b, c, A. If the correct value of A is 30°, find the percentage error in the calculated value of area of triangle. **5**

                OR

8. a) If $u^2 + xv^2 - uxy = 0$, $v^2 - xy^2 + 2uv + u^2 = 0$, find $\dfrac{\partial u}{\partial x}$ by choosing u , v as dependent and x, y as independent variables. **4**

   b) Show that $u = \dfrac{x+y}{1-xy}$, $v = \tan^{-1} x + \tan^{-1} y$ are functionally dependent and find the relation between them. **4**

   c) Find all the stationary values of the function f (x, y) = $x^3 + 3 xy^2 - 15 x^2 - 15 y^2 + 72 x$. Find maximum value of f(x, y) at suitable point. **5**

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