

Total No. of Questions—8]

[Total No. of Printed Pages—4+1

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[4856]-101

F.E. EXAMINATION, 2015
ENGINEERING MATHEMATICS—I
(2012 PATTERN)

Time : Two Hours

Maximum Marks : 50

- N.B. :—** (i) Attempt Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4,
Q. No. 5 or Q. No. 6, Q. No. 7 or Q. No. 8.
(ii) Neat diagrams must be drawn wherever necessary.
(iii) Figures to the right indicate full marks.
(iv) Use of logarithmic tables electronic pocket calculator is
allowed.
(v) Assume suitable data, if necessary.

1. (a) Examine for consistency of the system of equations : [4]

$$2x - 3y + 5z = 1$$

$$3x + y - z = 2$$

$$x + 4y - 6z = 1$$

if consistent solve it.

- (b) Find the eigenvalues of matrix : [4]

$$\begin{bmatrix} 1 & 0 & -4 \\ 0 & 5 & 4 \\ -4 & 4 & 3 \end{bmatrix}$$

hence find eigenvector corresponding to highest eigenvalue.

P.T.O.

- (c) Find the complex number z if $\text{amp}(z + 2i) = \frac{\pi}{4}$ and $\text{amp}(z - 2i) = \frac{3\pi}{4}$. [4]

Or

2. (a) Examine for the linear dependence or independence. If dependent, find the relation among the following vectors $(1, 1, 1), (1, 2, 3), (2, 3, 8)$. [4]
- (b) Find all values of $(1 + i)^{1/4}$. [4]
- (c) If $\sin(\alpha + i\beta) = x + iy$, then prove that : [4]

$$(i) \quad \frac{x^2}{\cosh^2 \beta} + \frac{y^2}{\sinh^2 \beta} = 1,$$

$$(ii) \quad \frac{x^2}{\sin^2 \alpha} - \frac{y^2}{\cos^2 \alpha} = 1.$$

3. (a) Test convergence of the series (any one) : [4]

$$(i) \quad \frac{1}{2+3} + \frac{2}{2+3^2} + \frac{3}{2+3^3} + \dots$$

$$(ii) \quad \sum_{n=1}^{\infty} \frac{\sqrt{n+1} + \sqrt{n}}{n^3}.$$

- (b) Prove that : [4]

$$e^{x \cos x} = 1 + x + \frac{x^2}{2} - \frac{x^3}{3} - \dots$$

- (c) Find n th derivative of [4]

$$\frac{x+1}{(x-1)(x+2)(x-3)}.$$

Or

4. (a) Solve any one : [4]

(i) $\lim_{x \rightarrow 0} (\cos x)^{1/x^2}$

(ii) $\lim_{x \rightarrow 0} \frac{e^{ax} - e^{-ax}}{\log(1 + bx)}$.

(b) Using Taylor's theorem, expand $x^4 - 5x^3 + 5x^2 + x + 2$ in powers of $x - 2$. [4]

(c) If $y = \cos(m \log x)$, then prove that : [4]

$$x^2 y_{n+2} + (2n + 1)xy_{n+1} + (m^2 + n^2)y_n = 0.$$

5. Solve any two :

(a) If [6]

$$u = \tan(y + ax) + (y - ax)^{3/2},$$

where a is a constant, then show that :

$$\frac{\partial^2 u}{\partial x^2} = a^2 \frac{\partial^2 u}{\partial y^2}.$$

(b) If [7]

$$u = x^3 f\left(\frac{y}{x}\right) + \frac{1}{y^3} \phi\left(\frac{x}{y}\right),$$

prove that :

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 9u.$$

(c) If [6]

$$u = f(x - y, y - z, z - x),$$

then find the value of :

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}.$$

Or

6. Solve any two :

(a) If $u = mx + ny$, $v = nx - my$, where m, n are constants, then find the value of : [6]

$$\left(\frac{\partial u}{\partial x}\right)_y \cdot \left(\frac{\partial y}{\partial v}\right)_x \cdot \left(\frac{\partial x}{\partial u}\right)_v \cdot \left(\frac{\partial v}{\partial y}\right)_u.$$

(b) If [7]

$$u = \tan^{-1} \left[\frac{x^3 + y^3}{x + y} \right],$$

then prove that :

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \sin 2u [1 - 4 \sin^2 u].$$

(c) If [6]

$$z = f(x, y), \quad x = \frac{\cos u}{v}, \quad y = \frac{\sin u}{v},$$

then prove that :

$$v \frac{\partial z}{\partial v} - \frac{\partial z}{\partial u} = (y - x) \frac{\partial z}{\partial x} - (y + x) \frac{\partial z}{\partial y}.$$

7. (a) If [4]

$$u = \frac{y - x}{1 + xy}, \quad v = \tan^{-1} y - \tan^{-1} x$$

find $\frac{\partial(u, v)}{\partial(x, y)}$.

(b) Prove that : [5]

$u = y + z$, $v = x + 2z^2$, $w = x - 4yz - 2y^2$,
are functionally dependent and find relation.

- (c) As dimensions of a triangle ABC are varied, show that the maximum value of $\cos A \cos B \cos C$ is obtained when the triangle is equilateral. [4]

Or

8. (a) If $u + v^2 = x$, $v + w^2 = y$, $w + u^2 = z$ find $\frac{\partial u}{\partial x}$. [4]
- (b) In estimating the cost of a pile of bricks measured 2 m \times 15 m \times 1.2 m, the top of the pile is stretched 1% beyond the standard length. If the count is 450 bricks in 1 cubic meter and bricks cost Rs. 450 per thousand, find the approximate error in cost. [5]
- (c) Find the minimum value of $x^2 + y^2$, subject to the condition $ax + by = c$. [4]